

## MAD 4401 ASSIGNMENT

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Work the following problems or give the correct statements. The assignment is due 1/16/15. Use the class periods on January 12 and 14 to work together on the problems and the statements of the theorems. Each person should turn in the assignment individually on Friday 1/16/15.

**Problem 1.** State the Intermediate Value Theorem

**Problem 2.** State the Mean Value Theorem.

**Problem 3.** Give the Taylor expansion of a function  $f(x)$  centered at  $a$ . Give this as a function of the derivatives of  $f(x)$  at  $a$ .

**Problem 4.** Suppose that  $h(x)$  is a continuous function. Suppose that  $\{x_0, h(x_0), h^2(x_0), \dots, h^n(x_0), \dots\} \rightarrow z$  as  $n \rightarrow \infty$ . Show that  $h(z) = z$ .

**Problem 5.** Suppose that  $p(x)$  is an odd degree polynomial. Show that there is a real number  $x$  such that  $p(x) = 0$ . Show that this may be false for an even degree polynomial.

**Problem 6.** Suppose that  $h(x)$  is differentiable on  $[a, b]$ . Suppose that  $h(z) = z$  and that  $|h'(z)| < 1$ . Show that there is an  $\varepsilon > 0$  such that for any  $x_0 \in (z - \varepsilon, z + \varepsilon)$ ,  $h^n(x_0) \rightarrow z$  as  $n \rightarrow \infty$ .

**Problem 7.** Suppose that  $f(x)$  is differentiable and that  $f(z) = 0$ . Let  $g(x) = x - \frac{f(x)}{f'(x)}$ . Show the following:

- (1)  $g(z) = z$ ,
- (2) the formula for the derivative of  $g(x)$ , and
- (3)  $g'(z) = 0$ .