MAD 4401 PRACTICE TEST 2

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Work all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. In Gambler's Ruin two players, A and B, are playing a game of chance with .55 being the probability that A would win and .45 the probability that B would win each play of the game. If A wins, he wins \$1.00 of B's money. If B wins, he wins \$1.00 of A's money. A has \$100.00 at the beginning of play and B has \$999,900.00. They continue to play until either A or B has all the money. What is the probability that A will win all the money? What is the probability that B will win all the money?

Problem 2. Consider the differential equation $\frac{dx}{dt} = t^2 \cdot x^2$ with initial condition $x(0) = 2$. Solve this differential equation numerically on $[0, 1]$ using the Runge Kutta method with $h = 1/20$ and $n = 10$. Give the estimated values of the points to eight digits.

Problem 3. Give the coefficients for estimating the third derivative of $f(x)$ at a using the points $\{a-2h, a-h, a+2h, a+3h, a+4h\}$. What would be the best choice of h to get the best estimate? What would be the error in the final estimate? Assume that the computer program will calculate to 20 digits of accuracy.

Problem 4. Determine the Taylor expansion for the solution of the differential equation $\frac{dx}{dt} = t^2 \cdot x$ with $x(0) = -1$. Determine the expansion to the t^6 term, $x(t) \approx a_0 + a_1 \cdot t + a_2 \cdot t^2 + a_3 \cdot t^3 + a_4 \cdot t^4 + a_5 \cdot t^5 + a_6 \cdot t^6$.

Problem 5. Derive the steady-state probabilities for a single server queue M/M/1/FIFO with service rate σ and arrival rate α assuming that $\sigma > \alpha > 0$.