MAD 4401 PRACTICE TEST 2

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Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. There are three urns, I, II, and III. Each urn has white balls and black balls. Urn I has 3W and 4B, Urn II has 7W and 1B. Urn III has 2W and 6B. An urn is chosen at random and a ball is chosen at random out of the urn. What is the probability that Urn II was chosen given that the ball was black?

Problem 2. Consider the differential equation $\frac{dx}{dt} = t^2 \cdot \sin(x)$ with initial condition $x(0) = 2$. Solve this differential equation numerically on $[0, 1]$ using the Runge Kutta method with $h = 1/20$ and $n = 10$. Give the estimated values of the points to eight digits.

Problem 3. Give the coefficients for estimating the second derivative of $f(x)$ at a using the points $\{a-3h, a-2h, a-h, a, a+h, a+2h, a+3h\}$. What would be the best choice of h to get the best estimate? What would be the error in the final estimate? Assume that the computer program will calculate to 60 digits of accuracy.

Problem 4. Determine the Taylor expansion for the solution of the differential equation $\frac{dx}{dt} = t^2 \cdot \sin(x)$ with $x(0) = \frac{\pi}{4}$. Determine the expansion to the t^{15} term, $x(t) \approx a_0 + a_1 \cdot t + a_2 \cdot t^2 + a_3 \cdot t^3 + a_4 \cdot t^4 + a_5 \cdot t^5 + a_6 \cdot t^6 + \cdots + a_{15} \cdot t^{15} + \cdots$.

Problem 5. Derive the steady-state probabilities for a queue with two servers M/M/2/FIFO with service rate σ and arrival rate α assuming that $2 \cdot \sigma > \alpha > 0$. Simulate such a queue with $\alpha = 9$ and $\sigma = 5$ for $t = 100$ and compare output with the steady-state probabilities.