MAD 4401 TEST 1 - JAMES KEESLING

Work all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. Solve the equation $x^3 + 5 = \cos x$ by the Newton-Raphson method. Give the Newton function. Find a starting point for which the method converges. Give the starting point and the iterations with twelve digits accuracy. Circle the final answer.

Problem 2. Let h be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 3. Determine the closed Newton-Cotes coefficients for eight intervals, $\{a_0, a_1, \ldots, a_8\}$.

Problem 4. Estimate $\int_0^\pi \sin(x^2)dx$ using Romberg Integration using 2^7 subintervals. Give the first column of the result and the last two columns to 12 digits. Circle the best answer.

Problem 5. Give the Legendre polynomials up to degree 10. Let $p(x)$ be the Legendre polynomial of degree 3. Show that $\int_{-1}^{1} p(x) \cdot q(x)dx = 0$ for for $q(x) = 1, q(x) = x$, and $q(x) = x^2$.