MAD 4401 TEST 1 - JAMES KEESLING

NAME _____

Work all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. Solve the equation $x^3 + 5 = \sin x^2$ by the Newton-Raphson method. Give the Newton function. Find a starting point for which the method converges. Give the starting point and the iterations with twelve digits accuracy. Circle the final answer.

Problem 2. Consider the points $\left\{x_0 = \frac{1}{4}, x_1 = \frac{2}{3}, x_2 = \frac{4}{5}\right\} \subset [0, 1]$. What should be the values of A_0, A_1, A_2 such that $\sum_{i=0}^{2} A_i \cdot f(x_i) = \int_0^1 f(x)dx$ is exact for $f(x)$ a polynomial of degree ≤ 2?

Problem 3. Give a formula to estimate the 3rd derivative of $f(x) = \cos(x^3)$ at $x = 2$ using the points $\{2-2h, 2-h, 2+h, 2+3h, 2+4h\}$. What is the best h to use and what is the error using this h? Assume that the computational error is $\varepsilon = 10^{-50}$.

Problem 4. Estimate $\int_0^1 \cos(x^3)dx$ using Romberg Integration using 2^7 subintervals. Give the first column of the result to five digits and the last two columns to 12 digits. Give the other columns to three digits. Circle the best answer.

Problem 5. Determine the points and weights to be used for Gaussian quadrature using 11 points. Give the Legendre polynomial of degree 11.