MAD 4401 TEST 2

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NAME: ________________________________

Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements that are incorrect.

Problem 1. In Gambler's Ruin two players, A and B, are playing a game of chance with .51 being the probability that A would win and .49 the probability that B would win each play of the game. If A wins, he wins $1.00 of B's money. If B wins, he wins $1.00 of A's money. A has $30.00 at the beginning of play and B has $170.00. They continue to play until either A or B has all the money. What is the probability that A will win all the money? What is the probability that B will win all the money?

Problem 2. Consider the differential equation $\frac{dx}{dt} = t \cdot x^2$ with initial condition $x(0) = -1$. Solve this differential equation numerically on $[0, 1]$ using the Runge Kutta method with $h = 1/5$ and $n = 5$. Give the calculated values of the points to eight digits.

Problem 3. Give the coefficients for estimating the second derivative of $f(x)$ at a using the points $\{a-2h, a-h, a+2h, a+3h, a+4h, a+5h\}$. What would be the best choice of h to get the best estimate? What would be the error in the final estimate? Assume that the computer will calculate $f(x)$ with error approximately 10^{-24} for x near a.

Problem 4. Determine the Taylor expansion for the solution of the differential equation $\frac{dx}{dt} = t \cdot x^3$ with $x(0) = 1$. Determine the expansion to the t^{10} term, $x(t) = a_0 + a_1 \cdot t + a_2 \cdot t^2 + a_3 \cdot t^3 + a_4 \cdot t^4 + \cdots + a_{10} \cdot t^{10} + \cdots$.

Problem 5. Derive the steady-state probabilities for a single server queue M/M/1/FIFO with service rate σ and arrival rate α assuming that $\sigma > \alpha$. Calculate the average number in the system.