MAD 4401 TEST 2 - JAMES KEESLING

NAME ____

Work all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. Solve the differential equation $\frac{dx}{dt} = t \cdot \sin(t \cdot x)$ with $x(0) = 3$. Use Runge-Kutta with $h = \frac{1}{10}$ and $n = 10$. Give the solution to eight digits. Estimate the local error and the global error.

Problem 2. Consider the differential equation $\frac{dx}{dt} = \cos(t \cdot x^2)$ with $x(0) = 2$. Determining the Taylor expansion of the solution to the t^8 term, $a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7 + a_8 t^8$.

Problem 3. In a medical test for a particular disease, the probability of the test being positive given that the subject has the disease is .99. The probability that the test is negative in this case is .01. The probability that test is negative given that the subject does not have the disease is .97. The probability of a positive result in this case is .03. Suppose that the probability of the disease is $\frac{1}{50{,}000}$. Suppose that a random individual is tested and the result of the test is positive. What is the probability that the individual has the disease?

Problem 4. Derive the formula for the steady-state probabilities for a queue with one server of the form M/M/1/FIFO. Assume that the arrival rate is α and the service rate is σ with $\sigma > \alpha$. Use the **queue** program to simulate one hundred days for a queue of the form M/M/1/FIFO with $\alpha = 4$ per day and $\sigma = 5$ per day. What proportion of the time was the queue in each of the states $n = 0, 1, 2$?

Problem 5. Gambler A has \$50. Gambler B has $\$999,950$. They play a game in which A wins with probability $p = .51$ and B wins with probability $q = .49$. The player winning a play receives \$1.00 from the other. They continue play until one player has all the money. What is the probability that A will win all of B's money?