THE DYADIC SOLENOID

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In this document we introduce the dyadic solenoid. There are several ways to produce a solenoid. Here we will be using the fact that in a Hausdorff space the nested intersection of a collection of compact connected sets is compact and connected.

Theorem. *Suppose that $A = \cap_{i=1}^{\infty} T_i$ where T_i is a compact connected set in X, then A is compact and connected.*

Let $F : T_0 \to T_1$ be a homeomorphism from a torus to another torus inside the first torus so that T_1 wraps twice around the center of T_0. The following figures show how this is done.

The dyadic solenoid is defined by

$$\Sigma_2 = \bigcap_{i=1}^{\infty} F^i(T_0).$$

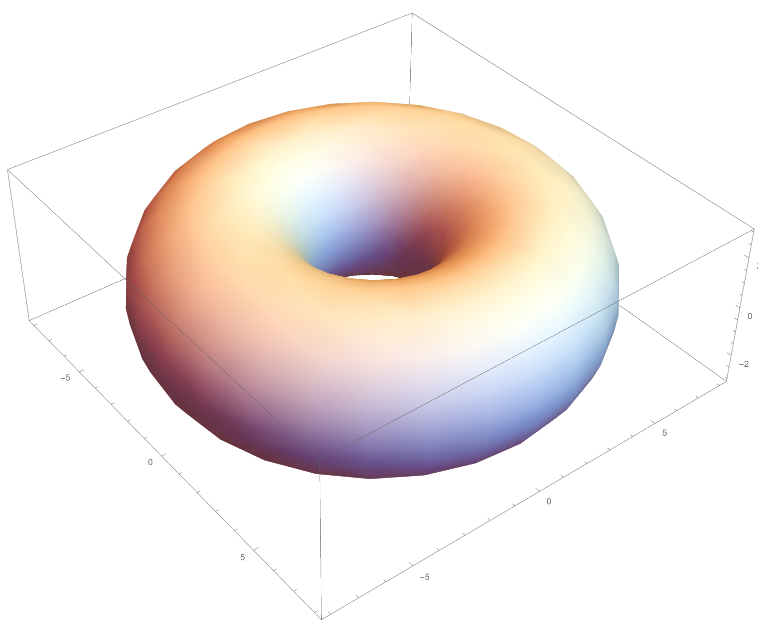

FIGURE 1. The torus T_0

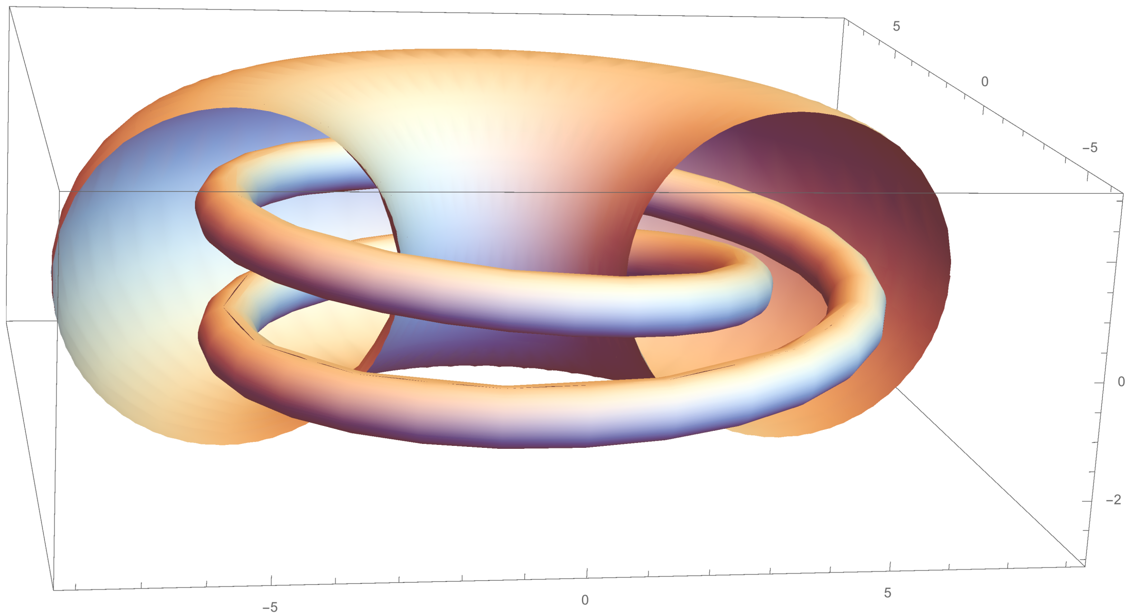

FIGURE 2. The torus $T_1 = F(T_0) \subset T_0$

This space is connected and compact by the Theorem. On the other hand, it will be shown in class that not every pair of points in the solenoid can be connected by an arc. The solenoid has some interesting properties. It is an example of a compact connected topological group.

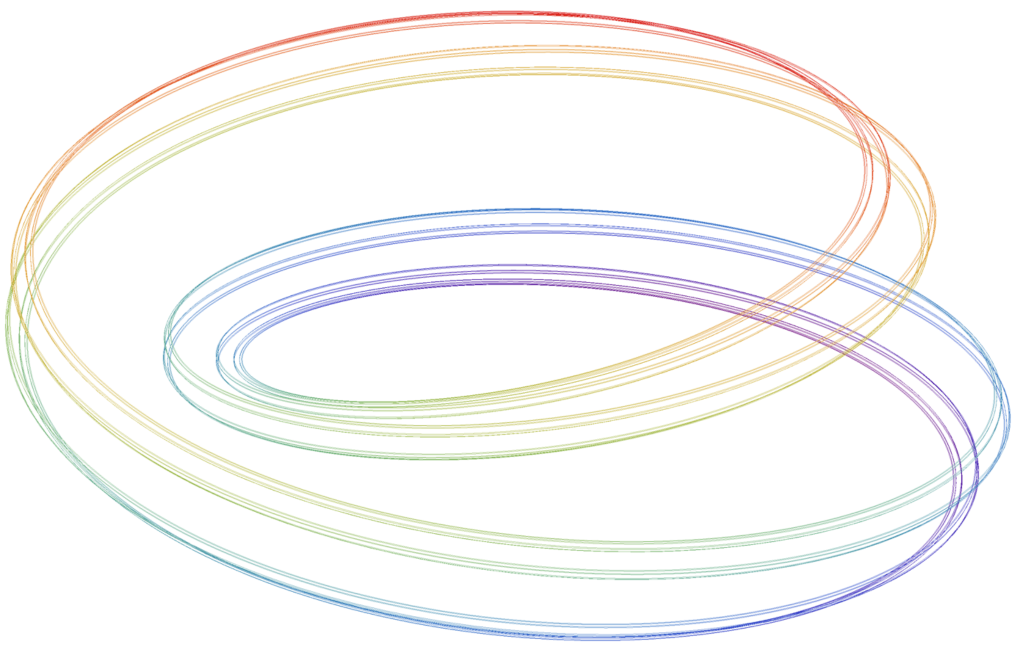

FIGURE 3. The solenoid $\Sigma_2 = \bigcap_{i=1}^{\infty} F^i(T_0)$