MAA 4212 – QUIZ 2 – JAMES KEESLING

Problem 1. State **Cavalieri's Principle**. Determine the volume of a sphere of radius R using this principle.

Problem 2. Use Pappus' Theorem to determine the surface area of a torus. Assume that the torus is formed by rotating a circle around the $z-$axis where the circle has radius b and the center of the circle is distance a from the $z-$axis with $a > b$.

Problem 3. Determine a formula for $\sum_{i=1}^{n} i^5$. Use this to determine the area under the curve of x^5 over $[0, 1]$.

Problem 4. State the **Fundamental Theorem of Calculus**.

Problem 5. State the definition of uniform convergence for a sequence of functions. Suppose that $f_n(x)$ is Riemann integrable for all n and that $f(x)$ is Riemann integrable. Show that if $f_n(x)$ converges to $f(x)$ uniformly on $[a, b]$, then $\lim_{n\to\infty} \int_a^b f_n(x) = \int_a^b f(x)dx$

Problem 6. State **Pappus' Theorem**. Prove Pappus' Theorem.

Problem 7. Show that for all $|x| < 1$ $\ln(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot x^n}{n}$.

Problem 8. Define the **Dirichlet Function** and show that it is not Riemann integrable over $[0, 1]$.

Problem 9. Let $f(x)$ be a bounded function over the interval $[a, b]$. Define when $f(x)$ is **Riemann integrable**. Show that if $f(x)$ is monotone increasing over the interval $[a, b]$, then $f(x)$ is Riemann integrable over $[a, b]$.

Problem 10. Estimate the integral of $f(x) = \exp(x^2)$ over the interval $[0, 1]$ using **Romberg Integration**. Use 2^5 intervals and determine the accuracy of your estimate.