MAD 4401 PRACTICE TEST 1 - SPRING 2018 - JAMES KEESLING

The problems that follow illustrate the methods covered in class. They are typical of the types of problems that will be on Test 1.

Problem 1. Determine a solution of the equation $x^5 - 3x^4 + 2x^3 - x^2 + x = 3$ using the Bisection method.

Problem 2. Determine a solution of $x = \cos x$ by the Newton-Raphson method.

Problem 3. Let h be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 4. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and that $f(z) = 0$. Suppose that $f'(z) \neq 0$. Let $g(x) = x - \frac{f(x)}{f'(x)}$ and suppose that $g'(x)$ is continuous at z. Show that there is an $\varepsilon > 0$ such that for any $x_0 \in [z - \varepsilon, z + \varepsilon]$, $g^n(x_0) \to z$ as $n \to \infty$

Problem 5. Determine the polynomial $p(x)$ of degree 5 passing through the points $\left\{(0,0), \left(\frac{1}{2}, 0\right), (1,0), \left(\frac{3}{2}, 1\right), (2,0), \left(\frac{5}{2}, 0\right)\right\}$.

Problem 6. Determine the closed Newton-Cotes coefficients for eleven points, $\{a_0, a_1, \ldots, a_{10}\}$ for the interval $[0,1]$. Use these values to estimate the integral $\int_{-4}^{4} \frac{1}{1+x^2} dx$.

Problem 7. Estimate $\int_0^{\sqrt{\pi}} \sin(x^2) dx$ using Gaussian quadrature using 8 points and using 15 points.

Problem 8. Estimate the integral $\int_0^1 \exp(x^2) dx$ using Romberg integration. Use the method up to 32 subdivisions of the interval. How many digits do you expect to be correct in the estimate?

Problem 9. Give the Legendre polynomials up to degree 6. List the properties that determine these polynomials.

Problem 10. Determine the coefficients to compute the first derivative of $f(x) = \sin(x^2)$ at $a = 2$ using the points $\{a-2h, a-h, a, a+h, a+2h\}$. Give the estimate of the derivative as a function of h.