MTG 5317/4303 FINAL - JAMES KEESLING

NAME ____

Work all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. Show that the Sorgenfrey line is Lindelöf.

Problem 2. Let $I = [0,1]$ be the interval. Show that there is a continuous function $f : I \to [0,1]^2$ which is onto.

Problem 3. Let $f, g : X \to \mathbb{R}^n$ be continuous. Show that $f(x)$ and $g(x)$ are homotopic.

Problem 4. Let (X, x_0) and (Y, y_0) be pointed spaces. Let $f : (X, x_0) \to (Y, y_0)$ be a continuous function. Define the homomorphism $f_* : \pi_1(X, x_0) \to \pi_1(Y, y_0)$.

Problem 5. State the following theorems.

The Urysohn Metrization Theorem

The Urysohn Lemma

The Tietze Extension Theorem

The Hahn-Mazurkiewicz Theorem

Problem 6. Show that $\pi_1(S^n) = 1$ for $n \geq 2$.

Problem 7. Show that $\pi_1(S^1, 1) \cong \mathbb{Z}$.

Problem 8. Note that $\pi_1(\mathbb{T}^2, 1) \cong \mathbb{Z}^2$. Find a continuous function $f : \mathbb{T}^2 \to \mathbb{T}^2$ such that $f_* = M : \pi_1(\mathbb{T}^2, 1) \to \pi_1(\mathbb{T}^2, 1)$ where

$$M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}.$$

Problem 9. Prove the Brouwer Fixed Point Theorem for D^2.

Problem 10. State the following theorems.

The Seifert-van Kampen Theorem

The Fundamental Theorem of Algebra

The Brouwer Fixed Point Theorem

The Jordan Curve Theorem