MGF1107 Homework 1

1. Recall the definitions of the following logical fallacies discussed in class.
   
   (a) Straw man: Constructing an argument concerning a distorted version of a statement, $P$, and using that argument to make a conclusion regarding $P$.
   
   (b) Circular reasoning: Using a statement, $P$, possibly restated, to conclude that $P$ is true.

   Explain why the following are logical fallacies and identify the type of fallacy.

   (i) The mayor wants to raise taxes to fund social programs, so she must not believe in the value of hard work.
   
   (ii) The state has no right to take a life, so the death penalty should be abolished

2. Recall that two formulas are equivalent if they have identical truth values. Equivalence can be shown by comparing truth tables.

   (a) For the statement $P$, use a truth table to show that $\neg\neg P$ is equivalent to $P$. ($\neg$ is the negation symbol, so if $P$ is true, then $\neg P$ is false.)

   (b) For statements $P$ and $Q$, use a truth table to show that $P \rightarrow Q$ is equivalent to $(\neg P) \lor Q$. ($\lor$ is 'or').

   (c) Recall that $\land$ is 'and'. Use a truth table to show that $\neg(P \land Q)$ is equivalent to $(\neg P) \lor (\neg Q)$.

   (d) Use the previous parts and the following

   
   \[ P \land Q = Q \land P \quad \text{and} \quad P \lor Q = Q \lor P \]

   to show that $P \rightarrow Q$ is equivalent to $(\neg Q) \rightarrow (\neg P)$

3. Recall the set notation we discussed in class of the form

   \[ \{ x \mid x \text{ has property } P \} \]

   Write the following using that notation: The set of integers greater than 5 and less than 9.

4. Let $A = \{ x \mid x < 6 \}$ and $B = \{ x \mid x < 15 \}$. 

(a) Find the intersection of $A$ and $B$, $A \cap B$
(b) Find the union of $A$ and $B$, $A \cup B$
(c) Find $B$ set minus $A$, $A \setminus B$

5. Let $A, B,$ and $C$ be sets. Use Venn diagrams to illustrate that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \text{ and } A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$