MGF1107 Homework 2

1. Reduce the following

   (a) 18 mod 3

   (b) -4 mod 5

   (c) -15 mod 60

   (d) 80 mod 12

2. Fill in the following tables by performing the indicated operation with each number in mod 6.

+	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

x	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

3. In the previous problem, the multiplication table should contain zeroes where two nonzero numbers are multiplied. These numbers are called *zero divisors*.

   (a) List all the zero divisors from the previous problem.

   (b) Do the integers mod 5, $\mathbb{Z}$ mod 5 have any zero divisors? What about $\mathbb{Z}$ mod 7, $\mathbb{Z}$ mod 3, and $\mathbb{Z}$ mod 4?

   (c) Can you see any condition on n that determines whether or not there will be zero divisors in $\mathbb{Z}$ mod n?

4. Let $A = \{0, 1, 2\}$ and $B = \{0, 1, 4\}$.

   (a) What is $A \times B$?

   (b) List the elements of the relation $R = \{(a, b) \in A \times B \mid b = a^2\}$?

5. Let C be the set of all cities, and N be the set of all countries. Describe, in set notation (as in the previous problem, part (b), the relation "c is a city located in the country n."

6. Let $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7, 8\}$, and $C = \{9, 10, 11, 12\}$

   Let $R = \{(1, 5), (1, 8), (3, 6), (2, 7)\}$ be a relation from A to B. Let $S = \{(5, 9), (8, 9), (6, 11), (7, 12)\}$ be a relation from B to C.

   (i) What is $S \circ R$?

   (ii) What is $R^{-1} \circ S^{-1}$?

   (iii) What is $(S \circ R)^{-1}$

   (iv) Which of the 5 relations listed in this question are functions? Explain your answer.

7. Suppose R is a relation from A to B and S and T are relations from B to C. Must the following statements be true? Justify your answers with proofs or counterexamples.

   (a) If $S \subseteq T$ then $S \circ R \subseteq T \circ R$.

   (b) $(S \cap T) \circ R \subseteq (S \circ R) \cap (T \circ R)$

   (c) $(S \cap T) \circ R = (S \circ R) \cap (T \circ R)$

   (d) $(S \cup T) \circ R = (S \circ R) \cup (T \circ R)$

8. Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$

   (a) Is $f = \{(1, 5), (2, 4), (3, 5)\}$ a function from A to B?

   (b) Is $f = \{(1, 5), (2, 4), (1, 6)\}$ a function from A to B?

9. Let $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. How many possible functions are there from A to B? How many possible functions are there from B to A?

10. Let $A = \{1, 2, 3, 4\}$ and $B = \{5, 6\}$. How many possible functions are there from A to B? How many possible functions are there from B to A?

11. Circle the figures below that are functions, and if they are not functions explain why.