MGF1107 Homework 3

1. Show whether or not the following sets form a group with the given operation. **Provide an explanation to justify your answers.**

    (a) $\mathbb{Z}/n\mathbb{Z}$, the integers modulo n, with addition

    (b) $\mathbb{Z}$, the integers, with multiplication

    (c) D_8, the dihedral group of order 8. This is the group of rotations and reflections of a square.

    (d) D_4, the dihedral group of order 4.

    (e) $\mathbb{Q}$, the rationals, with multiplication.

    (f) S_3, the set of permutations of the set $\{1, 2, 3\}$, with function composition.

2. What does it mean for a group, G, to be abelian?

3. Which of the groups from problem 1 are abelian?

4. Define the order of a finite group, G. Define the order of an element, $g \in G$.

5. Show that if G is a finite group, then for any $g \in G$, there is some integer, N, such that $g^N = 1$ (1 is the identity).

6. What does it mean for a group, G, to be cyclic?

7. Let G be a group, $g \in G$. If there exists $x, y \in G$ such that

$$gx = xg = 1 \text{ and } gy = yg = 1$$

What can you say about the relationship between x and y?