MGF1107 Exam 2

1 Symmetry

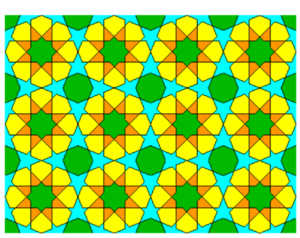
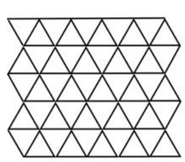
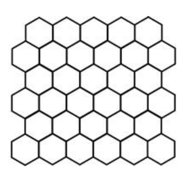
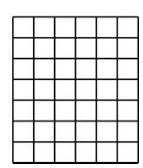
Symmetry is a property of an object that remains unchanged under certain operations. For example, the dihedral group, rotations and reflections of a regular polygon, is a group of symmetries of a regular polygon. Each rotation and reflection leaves the polygon unchanged.

There are three common symmetries associated with objects. We have

1. **Reflection symmetry** is when an object remains unchanged when reflected across a line.
2. **Rotation symmetry** is when an object remains unchanged after being rotated.
3. **Translation symmetry** is when a pattern remains unchanged when shifted.

2 Tilings

Tiling is an arrangement of polygons. It is a form of art that involves covering a flat area with geometric shapes. Tilings usually have regular or symmetric patterns. Examples of tiling are mosaics and brick walkways or walls. Some tiling is shown below.

Note that not every regular polygon can be exclusively used to create a tiling.

Theorem 1. *Tilings with a single regular polygon are possible only with equilateral triangles, squares, and hexagons.*

3 Proportion and the Golden Ratio

Suppose a rectangle has height L and width 1. The ratio of the height to width is $L/1$. Then section off the rectangle into a square with side lengths 1, and a rectangle with height $L-1$. There is a unique L such that the smaller rectangle has the same ratio of sides as the original, large rectangle. Mathematically,

$$\frac{L}{1} = \frac{1}{L-1}$$

This unique L is called the golden ratio, ϕ. Solving the above equation for L gives

$$\phi = \frac{1+\sqrt{5}}{2}$$

Let $(F_n)_{n\geq 0}$ be the sequence defined by the recurrence relation

$$F_{n+2} = F_{n+1} + F_n \text{ for } n > 1$$

and

$$F_0 = F_1 = 1$$

This gives the sequence

$$\{1, 1, 2, 3, 5, 8, 13, ...\}$$

The sequence (F_n) is the Fibonacci sequence.

There is a relationship between the Fibonacci numbers and the golden ratio. As n increases, the ratio of successive Fibonacci numbers approaches the golden ratio. We say the limit of the ratio of successive Fibonacci numbers, F_n/F_{n-1}, approaches ϕ as n goes to infinity. It is written

$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = \phi$$

.

Also, the Fibonacci numbers have a closed form expression involving the golden ratio. We have that each Fibonacci number is given by the following:

$$F_n = \frac{\phi^n - (1-\phi)^n}{\sqrt{5}}$$