University of Florida

MMB | EXAM I | DUE: JAN. 31, 2007

Name:

ID #:

Instructor:

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books but you can only use my help. I reserve the right to subtract points for my help.

Problem	Possible	Points
1	30	
2	30	
3	20	
4	20	
Total	100	

(1) (30 points) The table below shows the metabolism of phenytoin over time following an intravenous injection. Let $C(t)$ is the concentration of phenytoin in mg/L, and t is time in minutes after the injection.

Minutes	mg/liter
0	7.88
6	7.10
12	6.79
24	5.27
36	4.52
48	3.43
72	1.97
96	1.01
144	0.23

(a) Plot the data for $-\frac{dC}{dt}$. Approximate the derivative in the middle points with central difference, and the derivative at the endpoints with forward and backward difference. Fit a function through the given points (it will not be a line).

(b) Write a differential equation that will approximate the data. Use general constants for the parameters. Explain the meaning of the parameters. Estimate the parameters from the fit to the data in part (a).

(c) Plot the solution of the differential equation and the data on the same plot. What is the error, given by the sum of the squares of the differences of the y-coordinates of the data points and the points on the solution?

(2) (30 points) The data for the count of the US population are given on the last page. Suppose that the US population is described by the linear non-autonomous equation

$$N'(t) = r(t)N(t), \qquad N(0) = N_0$$

where time t is counted in years after 1790 ($t = 0$ is year 1790). The solution to this equation is

$$N(t) = N_0 e^{\int_0^t r(s)ds}.$$

Taking a ln of both sides we have

$$\ln N(t) = \ln N_0 + F(t) \qquad F(t) = \int_0^t r(s)ds.$$

(a) Plot the data of $\ln N$ against t.

(b) Fit to the best of your abilities a nonlinear function through the data in part (a). (You may want to try a parabola.) Determine N_0 and $r(t)$.

(c) Plot the solution of the differential equation and the data on the same plot. According to your model, what would the population of the US be in year 2010?

(3) (20 points) The following model was developed to describe the dynamics of the amount of vegetation $V(t)$ on a field of modest size where a herd of cows feeds on:

$$\frac{dV}{dt} = G(V) - Hc(V),$$

where $G(V) = rV(1-V/K)$ describes the growth of vegetation, r and K are positive constants; $c(V) = \beta V^2/(V_0^2 + V^2)$ is the consumption of vegetation per cow, β and V_0 are constants; H is the number of cows in the herd. Choose $r = 1/3$, $k = 25$, $\beta = 0.1$, $V_0 = 3$.

(a) Graph the functions $G(V)$ and $Hc(V)$ for different herd sizes $H = 10, 20, 30$. What conclusions can you draw by examining these graphs?

(b) Graph $\frac{dV}{dt}$ versus V for the same three herd sizes as in part (a). Use these graphs to determine all possible equilibria and their local stabilities. Justify your answers.

(c) For $H = 20$ graph two different solutions whose only differences are in the initial conditions and whose limit as time goes to infinity is different. This phenomenon is called *bistability*.

(4) (20 points) A fishery model with harvesting has the form

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H\frac{N}{A+N}.$$

This equation can be written in dimensionless form as

(1)
$$\frac{dx}{dt} = x(1-x) - h\frac{x}{a+x}.$$

(a) Show (by a graphical example) that depending on the parameters a and h equation (1) can have one, two or three equilibria. Classify the stability of equilibria in each case.

(b) Draw a bifurcation diagram of the equilibria x^* in terms of h for the cases $0 < a < 1$ and $a > 1$.

Appendix A.
United States' Population and Census Cost

Census year	Population	Census cost
1790	3,929,214	$44,377
1800	5,308,483	66,109
1810	7,239,881	178,445
1820	9,633,822	208,526
1830	12,866,020	378,545
1840	17,069,458	833,371
1850	23,191,876	1,423,351
1860	31,443,321	1,969,377
1870	38,558,371	3,421,198
1880	50,155,783	5,790,678
1890	62,979,766	11,547,127
1900	76,303,387	11,854,000
1910	91,972,266	15,968,000
1920	105,710,620	25,117,000
1930	122,775,046	40,156,000
1940	131,669,275	67,527,000
1950	151,325,798	91,462,000
1960	179,323,175	127,934,000
1970	203,302,031	247,653,000
1980	226,542,199	1,078,488,000
1990	248,718,301	2,492,830,000
2000	281,421,906	4,500,000,000