University of Florida

MMB EXAM II DUE: MARCH 19, 2007

Name:

ID #:

Instructor:

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books but you can only use my help. I reserve the right to subtract points for my help.

Problem	Possible	Points
1	25	
2	25	
3	35	
4	15	
Total	100	

(1) (25 points) Gause performed experiments (in the 1930's) with *Paramecium aurelia* and *Saccharomyces exiguus*. *P. aurelia* is a protozoa that feeds on *S. exiguus*. Gaus obtained the following data.

Time	S. exiguus	P. aurelia
0	155	90
1	40	175
2	20	120
3	10	60
4	25	10
5	55	20
6	120	15
7	110	55
8	50	130
9	20	70
10	15	30
11	20	15
13	70	20
15	135	30
16	135	80
17	50	170
18	15	90
19	20	30

We will try to fit these data to a Lotka-Volterra model

$$\begin{aligned} N' &= rN - cNP \\ P' &= bNP - \mu P \end{aligned} \tag{1}$$

(a) Compute the averages of $N(t)$ and $P(t)$ from the data

$$\hat{N} = \frac{1}{T}\int_0^T N(t)dt \qquad \hat{P} = \frac{1}{T}\int_0^T P(t)dt$$

Assume that you are looking at one period from time $t = 1$ through time $t = 8$ in the data set (T will be 7). Use composite trapezoidal rule to approximately compute the integrals from the data. Determine the coordinates of the equilibrium.

(b) Discuss why you think this choice of T and the time frame in the data set when one period occurs has been made. Would you choose another period and/or time frame in the data set when one period occurs? Why?

(c) Gause estimated $r = 0.65$ and $\mu = 0.32$. Use these data and the equilibrium to compute the other two coefficients, b and c.

(2) (a) (25 points) A population is governed by the difference equation

$$x_{n+1} = x_n e^{3-x_n}.$$

Show that all equilibria are unstable.

(b) The population is to be stabilized by removing a fraction p ($0 < p < 1$) of the population in each time period after births and deaths have taken place, to give the model

$$x_{n+1} = (1-p)x_n e^{3-(1-p)x_n}.$$

For what values of p does the population have an asymptotically stable positive equilibrium?

(3) (35 points) Consider the predator-prey model with functional respose of type III.

(2)
$$\begin{aligned} N' &= rN\left(1-\tfrac{N}{K}\right) - \tfrac{cN^2}{a^2+N^2}P \\ P' &= \tfrac{bN^2}{a^2+N^2}P - \mu P \end{aligned}$$

(a) The above model can be written in a nondimensional form as

(3)
$$\begin{aligned} x' &= f(x)[g(x)-y] \\ y' &= \beta[f(x)-\alpha]y \end{aligned}$$

where $f(x) = \frac{x^2}{1+x^2}$ and $g(x) = \frac{(1-\gamma x)(1+x^2)}{x}$. Derive that nondimensional form.

(b) Graph the x- and y- nullclines. Consider all distinct cases.

(c) Determine all equilibria.

(d) Determine the local stability of equilibria. Does Hopf bifurcation occur?

(4) (15 points) Consider the following model of smoking.

$$P' = \mu N - \beta \frac{PS}{N} - \mu P + \gamma S$$
$$S' = \beta \frac{PS}{N} - (\mu + \gamma) S \tag{4}$$

where $P(t)$ is the number of potential smokers (those that do not smoke but may start smoking) and $S(t)$ is the number of smokers. μ is the natural death rate, β is the rate at which non-smokers become smokers, and γ is the rate at which smokers become non-smokers. N is the total population size, that is $N(t) = P(t) + S(t)$.

(a) Show that $N(t)$ is in fact a constant. (Hint: What differential equation does $N(t)$ satisfy?)

(b) Show that the model about smoking has no periodic orbits in the first quadrant.