University of Florida

MAD4401 EXAM I October 3, 2003

Name:

ID #:

Instructor:

Directions: You have 50 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may not use a calculator.

Problem	Possible	Points
1	25	
2	15	
3	15	
4	5	
5	10	
6	30	
Total	100	

(1) Let $f(x) = x^5$.

(a) (15 points) Compute the fifth divided difference $f[0, 0, 1, 1, 1, 2]$ by completing the table below

x	f(x)	I DD	II DD	I DD	III DD	V DD

(b) (5 points) Write down the Hermite polynomial interpolating $f(x)$ at the points 0,0,1,1,1,2.

(c) (5 points) It is known that the fourth divided difference is expressible in terms of the fourth derivative of f:

$$f[0, 0, 1, 1, 1] = \frac{f^{(4)}(\xi)}{4!}$$

where $0 < \xi < 1$. Determine ξ.

(2) Let $f(x) = \dfrac{1}{x}$.

(a) (5 points) The Lagrange polynomial of degree n interpolating $f(x)$ at the points $x_0, \ldots, x_n$ is given by

$$P_n(x) = f(x_0)L_0(x) + \cdots + f(x_n)L_n(x)$$

where the basic Lagrange polynomials are

$$L_k(x) = \frac{(x - x_0)\ldots(x - x_{k-1})(x - x_{k+1})\ldots(x - x_n)}{(x_k - x_0)\ldots(x_k - x_{k-1})(x_k - x_{k+1})\ldots(x_k - x_n)}.$$

Write down the linear Lagrange polynomial $P_1(x)$ interpolating $f(x) = \frac{1}{x}$ at the points 1,2.

(b) (10 points) The error in the linear interpolation of f at the points 1,2 is known to be

$$E(x; f) = f(x) - P_1(x) = (x - 1)(x - 2)\frac{f''(\xi(x))}{2}$$

where $1 < \xi < 2$. If $f(x) = \frac{1}{x}$ find the smallest upper bound for the absolute value of the error $|E(x; f)|$. (Simplify your answer as much as possible.)

(3) (15 points) Find the decimal number that corresponds to the machine number

$$1 \qquad 00000010101 \qquad 000\ldots001$$

where $000\ldots001$ consists of 51 zeroes and 1 at the end.
Hint: Use the formula: $(-1)^s 2^{c-1023}(1+f)$. Find s, c and f.

(4) (5 points) Use chopping to approximate 0.0013551 to 3 significant digits.

(5) (10 points) Find the binary number that corresponds to the decimal number $\frac{1}{7}$.

(6) Consider the nonlinear equation $3x - \cos x = 0$ which can be rewritten as a fixed point problem in the form $x = \frac{1}{3}\cos x$.

(a) (5 points) Show graphically that this equation has a unique solution in the interval $[0, \frac{\pi}{2}]$.

(b) (10 points) If you are using the bisection method to solve the equation what is the smallest number of iterations needed to obtain the solution within 10^{-4}? Recall that for bisection method $|p_n - p| \leq \frac{b-a}{2^n}$.

(c) (10 points) Without computing the sequence $p_0, p_1 \ldots$ show that the iteration

$$p_{n+1} = \frac{1}{3}\cos p_n$$

converges for every p_0 in $[0, \frac{\pi}{2}]$.

(d) (5 points) What is the rate of convergence of the fixed point iteration in part (c)?