University of Florida

MAD4401 EXAM II NOVEMBER 5, 2003

Name:

ID #:

Instructor:

Directions: You have 50 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may not use a calculator.

Problem	Possible	Points
1	15	
2	15	
3	15	
4	15	
5	10	
6	20	
7	10	
Total	100	

(1) (15 points) Use the most accurate three-point formula to determine the missing entries in table

x	f(x)	$f'(x)$
2	1.5	
2.5	2	
3	3	
3.5	4.5	

Hint: The three-point formulas are:

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)]$$

$$f'(x_0) = \frac{1}{2h}[-f(x_0 - h) + f(x_0 + h)]$$

$$f'(x_0) = \frac{1}{2h}[f(x_0 - 2h) - 4f(x_0 - h) + 3f(x_0)]$$

(2) Consider the integral

$$\int_0^4 \frac{1}{\sqrt{1+x}}\,dx$$

(a) (5 points) Does the composite trapezoidal rule underestimate or overestimate the integral? Why?

(b) (10 points) What is the smallest number of subdivisions that must be used so that the composite trapezoidal rule approximates the integral within 10^{-3}?

Note: The error in the composite trapezoidal rule is

$$E_n^T(f) = -\frac{b-a}{12}h^2 f''(\xi).$$

(3) A clamped cubic spline S is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + Bx + 2x^2 - 2x^3 & 0 \le x < 1 \\ S_1(x) = 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3 & 1 \le x \le 2 \end{cases}$$

(a) (10 points) Find the constants B and b.

(b) (5 points) Find $f'(0)$ and $f'(2)$.

(4) (15 points) Determine the constants a, b, c, γ so that the quadrature formula

$$\int_0^1 f(x)\,dx \approx af(0) + bf(1) + cf''(\gamma)$$

has maximum degree of exactness. What is the degree of exactness of this formula?

(5) (10 points) For the initial value problem

$$
\begin{aligned}
&y' = f(t, y) \qquad\qquad 0 \leq t \leq 1 \\
&y(0) = 1
\end{aligned}
$$

show that the function $f(t, y) = e^t y + 1$ satisfies Lipschitz condition. What is the Lipschitz constant?

(6) (20 points) Compose Taylor's method of order two necessary for the solution of the following initial value problem

$$\begin{aligned} &y' = 1 + \frac{y}{t} \qquad && 1 \le t \le 2 \\ &y(1) = 2 \end{aligned}$$

with $h = 0.5$.

Hint: Taylor method of order 2 is given by

$$\begin{aligned} &w_0 = \alpha \\ &w_{i+1} = w_i + hT^{(2)}(t_i, w_i) \qquad && i = 0, 1, \ldots, N-1 \end{aligned}$$

where $T^{(2)}(t_i, w_i) = f(t_i, w_i) + \frac{h}{2} f'(t_i, w_i)$.

(7) (10 points) Compose Gaussian quadrature with $n = 3$ to estimate the integral

$$\int_0^{\pi} (\sin x)^2 dx$$

Do not simplify to a final answer. You will need the following formula for change of variables:

$$x = \frac{1}{2}[(b-a)t + a + b]$$

Hint: The roots and the coefficients are given in the table: