

University of Florida

MAP2302

FINAL EXAM

APRIL 29, 2004

Name:

ID #:

Instructor: Maia Martcheva

**Directions:** You have two hours to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use only a simple not graphic calculator.

Problem	Possible	Points
1	10	
2	30	
3	20	
4	20	
5	20	
6	20	
7	15	
8	25	
9	25	
10	15	
Total	200	

(1) (10 points) Determine the convergence set of the following power series

$$\sum_{n=0}^{\infty} \frac{n^2}{2^n} (x+2)^n$$

(2) (30 points) Solve the initial value problem

$$\left(\sqrt{\frac{y}{x}} + \cos x\right) \cdot dx + \left(\sqrt{\frac{x}{y}} + \sin y\right) \cdot dy = 0, \quad y(\pi) = \pi.$$

- (3) (25 points) Find the indicial equation and its roots for the specified singularity of the differential equation

$$x^2 y'' - (\sin x) y' - 4y = 0 \quad x = 0.$$

- (4) (20 points) Use Laplace transform to solve the given symbolic initial value problem

$$\begin{cases} y'' + y = \delta(t - 2\pi) \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$$

(5) (20 points) Find the inverse Laplace transform of

$$F(s) = \frac{se^{-s}}{s^2 + 4s + 13}.$$

(6) (20 points) Find the Laplace transform of

$$f(t) = \begin{cases} e^{-t}, & 0 < t < 1, \\ 1, & 1 < t < 2, \end{cases}$$

assuming that  $f(t)$  has period  $T = 2$ .

(7) (15 points) Find the Laplace transform of the given generalized function

$$\delta\left(t - \frac{\pi}{3}\right) e^{-t} \cos t.$$

(8) (20 points) Find the integrating factor of the equation

$$ydx + (2xy - e^{-2y})dy = 0$$

- (9) (30 points) Determine a Taylor's polynomial of degree three to approximate the solution of the following initial value problem

$$\begin{cases} y' = \sin y + e^x \\ y(0) = 0 \end{cases}$$

- (10) (15 points) Determine the form of the particular solution of the nonhomogeneous equation (do not evaluate the coefficients).

$$y'' + 2y' + 2 = e^{-x} \cos x$$