University of Florida

GNA HOMEWORK I Due: February 12, 2018

Name:

ID #:

Instructor: Maia Martcheva

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books and you may work together but each of you must submit a homework.

Problem	Possible	Points
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
Total	30	

(1) Given the following non-linear equation

$$x - \tan(x) = 0$$

(a) Determine an interval $[a, b]$ that contains the smallest positive root of the above equation.

(b) Write MATLAB codes that approximate this root to 10^{-7} using

- Bisection method
- Newton's method
- Secant method

(c) Present your results in a table that contains the first 2-3 iterates x_0, x_1, x_2 and the last few iterates x_{n-2}, x_{n-1}, x_n for each method.

(2) Consider the fixed point problem

$$x = \pi + 0.5\sin(x/2)$$

on the interval $[0, 2\pi]$.

(a) Show that the fixed point problem has a unique solution on that interval. These conditions also imply that the fixed point iteration will converge for any x_0 in the interval $[0, 2\pi]$.

(b) Use the estimate of the error to find the smallest number of iterations necessary to achieve accuracy of 10^{-2}.

(c) What is the rate of convergence of the fixed point iteration in part (a)?

(3) Let $p \geq 2$ be an integer. Consider Newton's method for finding the root $\sqrt[p]{a}$ for $a > 0$ by solving the equation $x^p - a = 0$. Assuming $x_0 > 0$ and $x_0^p \neq a$, derive the following results.

(a) Newton's method can be rewritten in the form

$$x_{n+1} = \frac{p-1}{p} x_n + \frac{a}{p x_n^{p-1}}$$

.

(b) $x_n > \sqrt[p]{a}$ for all $n \geq 1$.

(c) The iterates x_n are decreasing for $n \geq 1$.

(d) Show explicitly that this method is quadratic and find the asymptotic error constant.

(4) Given that the Newton iteration for finding a root r of f converges, $f \in C^2(R)$, and $f(r) = f'(r) = 0 \neq f''(r)$, prove that the convergence cannot be quadratic. Suggest a modification that restores quadratic convergence for smooth f.

(5) Show that

$$x_{n+1} = \frac{x_n(x_n^2 + 3a)}{3x_n^2 + a}, \qquad n \geq 0$$

is a third order method for computing $\sqrt{a}$, $a > 0$. Assuming $x_n \to \sqrt{a}$, find the asymptotic error constant.

(6) Consider the system

$$x = \frac{0.5}{1+(x+y)^2} \qquad y = \frac{0.5}{1+(x-y)^2}.$$

(a) Find a closed bounded and convex region $D \in R^2$ such that

(i) $g(D) \subset D$.

(ii) $\lambda = max_{x \in D} ||J(x)||_\infty < 1$

where $J(x)$ is the Jacobian of the system. Apply the appropriate Theorem to conclude that the system has a unque solution in D.

(b) Write MATLAB code that uses fixed point iteration to solve the system to within 10^{-7}. How many iterations did you need?