University of Florida

MMB HOMEWORK I DUE: FEB 1, 2012

Name:
ID #:
Instructor:

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books and you can work together but each of you must submit a homework.

Problem	Possible	Points
1	5	
2	5	
All	10	
Total	20	

(1) The table below shows the metabolism of phenytoin over time following an intravenous injection. Let $C(t)$ is the concentration of phenytoin in mg/L, and t is time in minutes after the injection.

Minutes	mg/liter
0	7.88
6	7.10
12	6.79
24	5.27
36	4.52
48	3.43
72	1.97
96	1.01
144	0.23

(a) Plot the data for $-\frac{dC}{dt}$ against C. Approximate the derivative with forward difference. Fit a function through the given points (it will not be a line).

(b) Write a differential equation that will approximate the data. Use general constants for the parameters. Explain the meaning of the parameters.

(c) Fit the model obtained in (b) to the data. Use the fit to estimate the parameters. Plot the solution of the differential equation and the data on the same plot. What is the error, given by the sum of the squares of the differences of the y-coordinates of the data points and the points on the solution?

(2) The following population model is often used as a simplified version of the logistic model: N'(t) $= \Lambda - \mu N(t)$.(0)

(a) Interpret the parameters of the model. What are their units?

(b) Assuming $N(0) = N_0$, solve the equation (2).

(c) Determine the equilibria of the model. What are their stabilities? Without using a graphing devise, graph some of the solutions.

(3) The following model was developed to describe the dynamics of the amount of vegetation $V(t)$ on a field of modest size where a herd of cows feeds on:

$$\frac{dV}{dt} = G(V) - Hc(V),$$

where $G(V) = rV(1-V/K)$ describes the growth of vegetation, r and K are positive constants; $c(V) = \beta V^2/(V_0^2 + V^2)$ is the consumption of vegetation per cow, β and V_0 are constants; H is the number of cows in the herd. Choose $r = 1/3$, $k = 25$, $\beta = 0.1$, $V_0 = 3$.

(a) Graph the functions $G(V)$ and $Hc(V)$ for different herd sizes $H = 10, 20, 30$. What conclusions can you draw by examining these graphs?

(b) Graph $\frac{dV}{dt}$ versus V for the same three herd sizes as in part (a). Use these graphs to determine all possible equilibria and their local stabilities. Justify your answers.

(c) For $H = 20$ graph two different solutions whose only differences are in the initial conditions and whose limit as time goes to infinity is different. This phenomenon is called *bistability*.

(4) Let the concentration of a substance in a cell be C and the concentration outside the cell be Γ. Assume that the substance enters the cell by diffusion at a rate βC. This would give the differential equation

$$C'(t) = \beta C(\Gamma - C)$$

where β is a constant. Now include an absorption term to give a differential equation

$$C'(t) = \beta C(\Gamma - C) - \frac{C}{1+C}.$$

(a) Find conditions on the parameters for the existence of 2 or 3 equilibria.

(b) Find the stabilities of the equilibria.

(c) Use computer algebra system to graph several representative solutions of the model with absorption. Graph C as a function of t.