University of Florida

GNA HOMEWORK II Due: March 14, 2018

Name:

ID #:

Instructor: Maia Martcheva

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books and you can work together but each of you must submit a homework.

Problem	Possible	Points
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
Total	30	

(1) Let $f(x) = x^5$.

(a) Compute the fifth divided difference $f[0, 0, 1, 1, 1, 2]$ by completing the table below

x	f(x)	I DD	II DD	I DD	III DD	V DD

(b) Write down the Hermite polynomial interpolating $f(x)$ at the points 0,0,1,1,1,2.

(c) It is known that the fourth divided difference is expressible in terms of the fourth derivative of f:

$$f[0, 0, 1, 1, 1] = \frac{f^{(4)}(\xi)}{4!}$$

where $0 < \xi < 1$. Determine ξ.

(2) For the basic Lagrange polynomials

$$L_k(x) = \Pi_{i\neq j}\frac{x - x_j}{x_i - x_j} \qquad \text{for} \qquad k = 0, \ldots, n,$$

show that

$$\sum_j L_j(x) = 1 \qquad \text{for} \quad \text{all} \qquad x.$$

(3) Let $x_0, x_1, \ldots, x_n$ be distinct real points, and consider the following interpolation problem. Choose a function

$$P_n(x) = \sum_{j=0}^{n} c_j e^{jx},$$

such that $P_n(x_i) = y_i$ for $i = 0, \ldots, n$.

(a) Show that this problem has a unique solution.

(b) What is the error of such interpolation?

(4) The following data is taken from a polynomial. Without deriving the polynomial itself, decide what its degree can be.

$$(x, p(x)) = \{(-2, -5), (-1, 1), (0, 1), (1, 1), (2, 7), (3, 25)\}.$$

(5) Consider the function e^x on $[0, b]$ and its approximation by an interpolating polynomial. For $n \geq 1$, let $h = b/n$, $x_j = jh$, $j = 0, 1, \ldots, n$, and let $P_n(x)$ be the nth-degree polynomial interpolating e^x on the nodes $x_o, \ldots, x_n$. Prove that as $n \to \infty$

$$\max_{x \in [0,b]} |e^x - P_n(x)| \to 0.$$

Hint: Show $|w_n(x)| \leq n! h^{n+1}$, for $0 \leq x \leq b$; look separately at each subinterval $[x_j, x_{j+1}]$.

(6) A fourth-degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24$, $\Delta^3 P(0) = 6$, and $\Delta^2 P(0) = 0$, where $\Delta P(x) = P(x+1) - P(x)$. Compute $\Delta^2 P(10)$.