University of Florida

BMS HOMEWORK III Due: March 16, 2016

Name:

ID #:

Instructor: Maia Martcheva

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books and you may work together. Each group should submit one homework.

Problem	Possible	Points
1	10	
2	10	
3	10	
Total	30	

(1) Consider the following model with mutation.

$$
\begin{aligned}
S' &= \Lambda - \frac{\beta_1 SI}{N} - \frac{\beta_1 SJ}{N} - \mu S, \\
I' &= \frac{\beta_1 SI}{N} - (\mu + \alpha_1 + m)I, \\
J' &= \frac{\beta_2 SJ}{N} - (\mu + \alpha_2)J + mI
\end{aligned} \tag{1}
$$

Assume $\mathcal{R}_2 < 1$. A vaccine is being designed but it may include only one of the strains. In that case the vaccine will be perfect with respect to the vaccine strain and not effective at all with respect to the other. Which of the strains should be the vaccine strain so that the vaccine eliminates both strains?

(2) Consider the following model of vaccination in a disease with vertical transmission:

$$\begin{aligned}\frac{dS}{dt} &= (1-p)\pi + (r_1 S + r_2 \eta I)\left(1 - \frac{S+I}{K}\right) - \beta S I - \mu S \\ \frac{dI}{dt} &= r_2(1-\eta)I\left(1 - \frac{S+I}{K}\right) + \beta S I - (\mu+\alpha)I \\ \frac{dV}{dt} &= p\pi - \mu V\end{aligned} \tag{2}$$

where the vaccine is applied at the entry point to the population and a fraction p is being vaccinated. r_1 and r_2 are the reproduction rates of susceptible and infected individuals respectively, η is the fraction of the progeny of infected individuals that are susceptible.

(a) Compute the disease-free equilibrium and the reproduction number $\mathcal{R}_0(p)$. Determine the stability of the disease-free equilibrium based on the reproduction number.

(b) Derive an equation for the endemic equilibrium. Show that backward bifurcation may occur, even though the vaccine is perfect.

(3) Consider the model with imperfect vaccination

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta SI - (\mu + \psi)S \\
\frac{dI}{dt} &= \beta SI + \sigma\beta VI - (\mu + \gamma)I \\
\frac{dV}{dt} &= \psi S - \sigma\beta VI - \mu V + \gamma I
\end{aligned} \tag{3}
$$

(a) Transform the problem into an optimal control problem.

(b) Use Fillipov-Cesari Theorem to prove that the optimal control problem has a solution.

(c) Apply Pontryagin's minimum principle to derive the optimal control and the system for adjoint variables.

(d) Write a MATLAB code to simulate the system with the control and without the control.