University of Florida

GNA HOMEWORK III Due: April 4, 2018

Name:

ID #:

Instructor: Maia Martcheva

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books and you can work together but each of you must submit a homework.

Problem	Possible	Points
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
Total	35	

(1) Solve the following minimization problems:

(a)

$$\int_{-1}^{1} (a + bx + x^2)^2 dx -> \min_{a,b}.$$

(b)

$$\max_{x \in [-1,1]} |a + bx + x^2| -> \min_{a,b}.$$

(2) Use the Gram-Schmidt method to determine the first three orthonormal polynomials on $[0, 1]$ with weight $w(x) = -\ln x$.

(3) Show that if $f(x)$ is even on $[-a, a]$, then its least squares approximation polynomial with weight $w(x) = 1$ is also even.

(4) Find the value α that minimizes

$$\int_0^1 |e^x - \alpha| dx.$$

(5) Show that the linear minimax approximation to $f(x) = \sqrt{1+x^2}$ on $[0,1]$ is

$$q_1^*(x) = .955 + .414x.$$

(6) Find a formula of the form $y = A \cdot 2^{Mx}$ for the approximation of the data

x	1	2	3	4	5
y	1	2	4	8	32

(7) Let $x_0, x_1, \ldots, x_n$ be distinct points in a finite interval $[a, b]$ and $f \in C^1[a, b]$. Show that for any $\epsilon > 0$ there exists a polynomial $p(x)$ such that $||f - p||_\infty < \epsilon$ and $p(x_i) = f(x_i)$ for all $i = 0, 1, \ldots, n$. Here, $|| \cdot ||_\infty$ denotes the sup norm on $[a, b]$.