University of Florida

MMB HOMEWORK III DUE: MARCH 16, 2012

Name:

ID #:

Instructor:

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books and you can work together but each of you must submit a homework.

Problem	Possible	Points
1	5	
2	5	
All	10	
Total	20	

(1) Population is modeled by the following integro-differential equation with distributed delay:

$$N'(t) = rN(t)\left(1 - \frac{1}{K}\int_0^\infty N(t-u)p(u)du\right). \tag{1}$$

where $p(u)$ is given by the following function

$$p(u) = \frac{1}{\tau}e^{-\frac{u}{\tau}}$$

which has an average delay

$$\int_0^\infty up(u)du = \tau.$$

This function $p(u)$ is called *weak generic delay kernel.*

(a) Show that all equilibria of model (1) are $N_1^* = 0$ and $N_2^* = K$.

(b) Show that the equation linearized about $N_2^* = K$ ($N(t) = N_2^* + v(t)$) is given by

$$v'(t) = -r\int_0^\infty v(t-u)p(u)du.$$

(c) This equation is linear so we expect solutions of the form $v(t) = \bar{v}e^{\lambda t}$. Derive the characteristic equation in λ.

(d) Show that the steady state N_2^* is locally asymptotically stable.

(2) For the following systems of linear equations, determine the stability and the type (spiral, node, etc.) of the $(0,0)$ equilibrium.

(a) System

$$\begin{aligned} x' &= 3x - 2y \\ y' &= 4x + y \end{aligned} \tag{2}$$

(b) System

$$\begin{aligned} x' &= 5x + 2y \\ y' &= -13x - 5y \end{aligned} \tag{3}$$

(c) System

$$\begin{aligned} x' &= -x - y \\ y' &= \tfrac{8}{5}x - y \end{aligned} \tag{4}$$

(3) Consider the following Lotka-Volterra predator-prey model

$$
\begin{aligned}
N' &= rN\left(1-\frac{N}{K}\right)-\frac{sNP}{1+shN} \\
P' &= \frac{esNP}{1+shN}-\mu P
\end{aligned}
\tag{5}
$$

(a) Determine the dimension of all parameters.

(b) Show that with the following change of variables $t = h\tau/e$, $u = N/K$ and $v = sP/r$ the model above can be reduced to the following dimensionless form

$$
\begin{aligned}
u' &= a\left(u(1-u)-\frac{uv}{1+bu}\right) \\
v' &= v\left(\frac{bu}{1+bu}-c\right)
\end{aligned}
\tag{6}
$$

where $a = rh/e$, $b = shK$, $c = \mu h/e$.

(c) Draw the x and y-nullclines of system (6) in the case when $c < 1$ and $b-c(b+1) > 0$, that is in the case when an interior equilibrium exists. You may use a computer algebra system. Indicate on the figure clearly where the equilibria are. Draw by hand the direction field on the nullclines. Justify your answer.

(d) Use a computer algebra system to draw the vector field together with several representative solutions of the model (6).

(4) Let a the dynamics of a predator-prey relationship with a refuge for the prey is given by the model in a non-dimensional form:

$$\begin{aligned} u' &= u - (u-k)v \\ v' &= av(u-k-1) \end{aligned} \tag{7}$$

where a and k are parameters.

(a) Find all nonnegative equilibria of model (7).

(b) Find a generic form of the Jacobian of model (7).

(c) Compute the Jacobian around each equilibrium.

(d) Determine the stability of each equilibrium.

(e) Use computer algebra system to graph a representative solution of model (7). Draw the solutions in the plane (u, v).