University of Florida

BMS HOMEWORK IV Due: November 18, 2015

Name:

ID #:

Instructor: Maia Martcheva

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books and you can work together but each of you must submit a homework.

Problem	Possible	Points
1	10	
2	10	
3	10	
4	10	
Total	40	

(1) Malaria is a vector-borne disease transmitted by mosquitoes. A percentage of the pregnant women who are infected with malaria give birth to malaria-infected newborns. We consider the model of vector-borne disease with temporary immunity and incorporate in it vertical transmission. The model for the vector is as before:

$$\begin{aligned} S_v' &= \mu_v - paS_vI - \mu_vS_v \\ I_v' &= paS_vI - \mu_vI_v \end{aligned} \tag{1}$$

where the total population size of the vector is $S_v + I_v = 1$. The model for the human population incorporates the vertical transmission:

$$\begin{aligned} S' &= \mu(S + \sigma I + R) - qaSI_v - \mu S + \gamma R \\ I' &= (1-\sigma)\mu I + qaSI_v - (\mu + \alpha)I \\ R' &= \alpha I - (\mu + \gamma)R. \end{aligned} \tag{2}$$

The model for humans also assumes that the total human population is $S + I + R = 1$. The new parameter σ gives the fraction of newborns to infected individuals who are healthy.

(a) Draw a flow-chart of the model (1)-(2).

(b) Use the Jacobian approach to compute the basic reproduction number of the model (1)-(2). Attempt to interpret $\mathcal{R}_0$ epidemiologically.

(c) Use the next-generation approach to compute the basic reproduction number of the model (1)-(2). Attempt to interpret $\mathcal{R}_0$ epidemiologically.

(2) Consider a model of HIV/AIDS with treatment and prevention:

$$
\begin{aligned}
S' &= \Lambda - (1-p)\frac{\beta_1 SI + \beta_2 ST + \beta_3 SA}{N} - \mu S \\
I' &= (1-p)\frac{\beta_1 SI + \beta_2 ST + \beta_3 SA}{N} - (\mu+\rho+\sigma)I \\
T' &= \rho I - (\mu+\gamma)T \\
A' &= \sigma I + \gamma T - (\mu+d)A
\end{aligned} \tag{3}
$$

where S is the number of susceptible individuals, I is the number of infected with HIV but not treated individuals, T is the number of infected with HIV but treated individuals, and A is the number of individuals with AIDS. The meanings of the parameters are as follows: p is the proportion protected by condom use, β_1,β_2 and β_3 are the transmission rates of the infected, treated and AIDS individuals respectively, ρ is the treatment rate, σ is the progression rate to AIDS without treatment, γ is the progression rate to AIDS with treatment, and d is the disease-induced death rate.

(a) Draw a flow-chart of the model.

(b) Use the Jacobian approach to determine the reproduction number and the stability of the disease-free equilibrium. Interpret the expression for $\mathcal{R}_0$ epidemiologically.

(c) Use van den Driessche and Watmough next-generation approach to compute the basic reproduction number. Interpret the expression for $\mathcal{R}_0$ epidemiologically, if different than above.

(d) Verify that $\mathcal{R}_0$ decreases with increasing p and ρ. Compute critical fraction of condom use p_c, so that $\mathcal{R}_0(p_c) = 1$. Plot p_c as a function of σ. Interpret what you observe epidemiologically.

(3) **Model with Quarantine, Isolation**

Given a model with quarantine and isolation.

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta\frac{S(I+qE)}{N-Q} - \rho S - \mu S + \eta_1 Q_1 \\
\frac{dE}{dt} &= \beta\frac{S(I+qE)}{N-Q} - \rho E - (\mu+\gamma)E \\
\frac{dQ_1}{dt} &= \rho S + \rho E - (\mu+\eta_1+\eta_2)Q_1 \\
\frac{dI}{dt} &= \gamma E - (\mu+\sigma+r_2)I \\
\frac{dQ_2}{dt} &= \sigma I + \eta_2 Q_1 - (\mu+r_1)Q_2 \\
\frac{dR}{dt} &= r_2 I + r_1 Q_2 - \mu R
\end{aligned}
$$

(a) Explain the meanings of the parameters.

(b) Draw a flowchart of the model.

(c) Use van den Driessche and Watmough next-generation approach to compute the basic reproduction number. Can you interpret the expression for $\mathcal{R}_0$ epidemiologically.

(d) Plot the reproduction number as a function of two variables: the quarantine rate and the isolation rate. What epidemiological conclusions can you draw from this plot?

(4) The new cases of malaria in India were slowly increasing in the second half of the 1980's. In 1997 India implemented new control strategies and the number of malaria cases have been decreasing ever since. The table below gives the India's population along with malaria cases for the period 1985-2011.

Year	Population	Number of cases	Year	Population	Number of Cases
1985	726	1.864	1999	948.66	2.28
1986	737	1.792	2000	970	2.032
1987	753.55	1.66	2001	984.58	2.085
1988	766.92	1.85	2002	1025.56	1.84
1989	769.32	2.05	2003	1027.16	1.87
1990	784.42	2.019	2004	1044.74	1.915
1991	808.1	2.12	2005	1007.2	1.82
1992	824.14	2.13	2006	1064	1.77
1993	833.89	2.21	2007	1089.8	1.51
1994	861.73	2.51	2008	982	1.53
1995	878.96	2.988	2009	943.93	1.56
1996	905.71	3.04	2010	1024.66	1.6
1997	884.72	2.66	2011	1059.8	1.31
1998	907.3	2.223	2012		

(a) Fit the logistic equation to India's population size:

$$P'(t) = rP(t)\left(1 - \frac{P(t)}{K}\right).$$

(b) Fit the following malaria model to the data:

$$\begin{aligned} C'(t) &= b(1 - \xi H(t-\tau))(P(t) - C(t) - I(t))y(t) - (\nu + \mu)C(t) \\ I'(t) &= \nu C(t) - (\lambda P(t) + \gamma P(t)H(t-\tau) + \mu)I(t) \\ y'(t) &= \rho(1 - \xi H(t-\tau))(1 - y(t))I(t) - (d + \eta H(t-\tau))y(t) \end{aligned} \tag{4}$$

where $C(t)$ is the number of symptomatic cases, $I(t)$ is the number of infectious individuals and $y(t)$ is the proportion of mosquitoes. The function $H(t-\tau)$ is the Heaviside function. Pre-estimate the value of τ to give mid-1997. In mid-1997 after the new measures are implemented b and ρ decrease while d increases. **Hint:** You must fit the incidence $b(1-\xi H(t-\tau))(P(t)-C(t)-I(t))y(t)$ to the number of new cases of malaria. It may be easier if you take ξ, η and γ equal to zero then fit the remaining parameters to the 1985-1996 data. Fix those parameters at their values and fit ξ, η and γ to the 1997-2011 data.