

University of Florida

GNA

HOMEWORK IV

DUE: APRIL 18, 2018

Name:

ID #:

Instructor: Maia Martcheva

Directions: You have until 5:00 p.m. on the due date to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use any books and you can work together but each of you must submit a homework.

Problem	Possible	Points
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
Total	30	

- (1) Derive an $\mathcal{O}(h^4)$ five-point formula to approximate $f'(x_0)$ that uses $f(x_0-h)$, $f(x_0)$, $f(x_0+h)$, $f(x_0+2h)$, and $f(x_0+3h)$.

Hint: Consider the expression $Af(x_0-h) + Bf(x_0+h) + Cf(x_0+2h) + Df(x_0+3h)$. Expand in four Taylor polynomials, and choose A, B, C, and D appropriately.

- (2) Estimate the number of subintervals required to obtain $\int_0^1 e^{-x^2} dx$ to 6 correct decimal places (absolute error $\leq 1/2 \times 10^{-6}$)
- (a) by means of the composite trapezoidal rule,
 - (b) by means of the composite Simpson's rule.

(3) Derive the two-point Gaussian quadrature formula for

$$I(f) = \int_0^1 f(x) \log(1/x) dx$$

in which the weight function is $w(x) = \log(1/x)$.

(4) Determine the quadrature formula of the type

$$\int_{-1}^1 f(t)dt = \alpha \int_{-1}^{-1/2} f(t)dt + \beta f(0) + \gamma \int_{1/2}^1 f(t)dt$$

having maximum degree of exactness d . What is the value of d ?

- (5) Let $a \leq x_0 < x_1 < \dots < x_n \leq b$ be an arbitrary fixed partition of the interval $[a, b]$. Show that there exist unique numbers $\gamma_0, \dots, \gamma_n$ with

$$\sum_{i=0}^n \gamma_i P(x_i) = \int_a^b P(x) dx$$

for all polynomials P with degree $(P) \leq n$.

Hint: Set $P(x) = 1, x, \dots, x^n$. Compare the resulting system of linear equations with that representing the polynomial interpolation problem with support abscissas $x_i, i = 0, \dots, n$.

(6) The integral

$$(f, g) = \int_{-1}^1 f(x)g(x)dx$$

defines a scalar product for functions $f, g \in C[-1, 1]$. Show that if f and g are polynomials of degree less than n , if x_i , $i = 1, 2, \dots, n$, are the roots of the n th Legendre polynomial, and if

$$\gamma_i = \int_{-1}^1 L_i(x)dx$$

where

$$L_i(x) = \prod_{k \neq i, k=1}^n \frac{x - x_k}{x_i - x_k} \quad i = 1, 2, \dots, n$$

then

$$(f, g) = \sum_{i=1}^n \gamma_i f(x_i)g(x_i).$$