Linearity: $\mathscr{L}\{af + bg\} = a\mathscr{L}\{f\} + b\mathscr{L}\{g\}$.

Translation in s: $\mathscr{L}\{e^{at}f(t)\}(s) = F(s-a)$, where $F = \mathscr{L}\{f\}$.

Translation in t: $\mathscr{L}\{g(t)u(t-a)\}(s) = e^{-as}\mathscr{L}\{g(t+a)\}(s)$, where $u(t-a)$ is the step function that equals 1 for $t > a$ and 0 for $t < a$. If $f(t)$ is continuous and $f(0) = 0$, then

$$\mathscr{L}^{-1}\{e^{-as}F(s)\}(t) = f(t-a)u(t-a) ,$$

where $f = \mathscr{L}^{-1}\{F\}$.

Convolution Property: $\mathscr{L}\{f * g\} = \mathscr{L}\{f\}\mathscr{L}\{g\}$, where $f * g$ denotes the convolution function

$$(f * g)(t) := \int_0^t f(t-v)g(v)\,dv .$$

$$\text{(2)} \qquad \mathscr{L}\{f^{(n)}\}(s) = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0) .$$

$$\mathscr{L}\{t^nf(t)\}(s) = (-1)^n\frac{d^nF}{ds^n}(s) , \quad \text{where} \quad F = \mathscr{L}\{f\} .$$

$$\mathscr{L}\{f\}(s) = \frac{\int_0^T e^{-st}f(t)\,dt}{1 - e^{-sT}} .$$

Variation of Parameters: $y(t) = v_1(t)y_1(t) + v_2(t)y_2(t)$.

where v_1' and v_2' are determined by the equations

$$v_1'y_1 + v_2'y_2 = 0$$
$$v_1'y_1' + v_2'y_2' = g(t)/a .$$

$$\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta)),$$
$$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha-\beta) + \sin(\alpha+\beta)),$$
$$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha-\beta) + \cos(\alpha+\beta)).$$

TABLE 7.1 BRIEF TABLE OF LAPLACE TRANSFORMS

$f(t)$	$F(s) = \mathscr{L}\{f\}(s)$
1	$\frac{1}{s}, \quad s > 0$
e^{at}	$\frac{1}{s-a}, \quad s > a$
$t^n, \quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
$\sin bt$	$\frac{b}{s^2+b^2}, \quad s > 0$
$\cos bt$	$\frac{s}{s^2+b^2}, \quad s > 0$
$e^{at}t^n, \quad n = 1, 2, \ldots$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$