PROOFS
LINEAR ALGEBRA

(1) (Due 8/29) Let $W$ be a subset of a vector space $V$ such that $W \neq \emptyset$ and for all $a \in F$ and $x, y \in W$, $ax + y \in W$. Prove that $W$ is a subspace of $V$.

(2) (Due 9/5) Let $S_1$ and $S_2$ be subsets of a vector space $V$. Prove that $\text{Span}(S_1 \cap S_2) \subseteq \text{Span}(S_1) \cap \text{Span}(S_2)$.

(3) (Due 9/19) Let $V$ be a vector space, and let $S_1 \subseteq S_2 \subseteq V$. Prove that if $S_2$ is linearly independent, then $S_1$ is linearly independent.

(4) (Due 9/26) Let $\{u, v, w\}$ be a basis for a vector space $V$. Prove that $\{u - w, v - w, w\}$ is also a basis for $V$.

(5) (Due 10/3) Let $T: \mathbb{R} \to \mathbb{R}$ be a linear transformation. Prove that there exists $m \in \mathbb{R}$ such that for all $x \in \mathbb{R}$, $T(x) = mx$.

(6) (Due 10/17) Let $V$ and $W$ be vector spaces, and let $T$ and $U$ be non-zero linear transformations from $V$ to $W$. If $\text{Range}(T) \cap \text{Range}(U) = \{0\}$, prove that $\{T, U\}$ is a linearly independent subset of $\mathcal{L}(V,W)$.

(7) (Due 10/24/17) Let $V$ be a finite dimensional vector space, and let $T: V \to V$ be a linear transformation which is not invertible. Prove that there exists a non-zero linear transformation $U: V \to V$ such that $UT$ is the zero transformation.

(8) (Due 10/31/17) Let $A \in \mathcal{M}_{m \times n}(F)$ and $B \in \mathcal{M}_{n \times m}(F)$. Suppose $n < m$. Prove that $AB$ is not invertible. Does the same result hold if $m < n$?