(1) (Due 8/29) Let $W$ be a subset of a vector space $V$ such that $W \neq \emptyset$ and for all $a \in F$ and $x, y \in W$, $ax + y \in W$. Prove that $W$ is a subspace of $V$.

(2) (Due 9/5) Let $S_1$ and $S_2$ be subsets of a vector space $V$. Prove that $\text{Span}(S_1 \cap S_2) \subseteq \text{Span}(S_1) \cap \text{Span}(S_2)$.

(3) (Due 9/19) Let $V$ be a vector space, and let $S_1 \subseteq S_2 \subseteq V$. Prove that if $S_2$ is linearly independent, then $S_1$ is linearly independent.

(4) (Due 9/26) Let $\{u, v, w\}$ be a basis for a vector space $V$. Prove that $\{u - w, v - w, w\}$ is also a basis for $V$.

(5) (Due 10/3) Let $T : \mathbb{R} \to \mathbb{R}$ be a linear transformation. Prove that there exists $m \in \mathbb{R}$ such that for all $x \in \mathbb{R}$, $T(x) = mx$.

(6) (Due 10/17) Let $V$ and $W$ be vector spaces, and let $T$ and $U$ be non-zero linear transformations from $V$ to $W$. If $\text{Range}(T) \cap \text{Range}(U) = \{0\}$, prove that $\{T, U\}$ is a linearly independent subset of $\mathcal{L}(V, W)$.

(7) (Due 10/24/17) Let $V$ be a finite dimensional vector space, and let $T : V \to V$ be a linear transformation which is not invertible. Prove that there exists a non-zero linear transformation $U : V \to V$ such that $UT$ is the zero transformation.

(8) (Due 10/31/17) Let $A \in \mathcal{M}_{m \times n}(F)$ and $B \in \mathcal{M}_{n \times m}(F)$. Suppose $n < m$. Prove that $AB$ is not invertible. Does the same result hold if $m < n$?
(9) (Due 11/7/17) Let $\delta : \mathcal{M}_{2 \times 2}(F) \to F$ be a function such that $\delta(I_2) = 1$ and $\delta$ satisfies the following conditions with respect to elementary row operations:

(a) If $A \rightarrow_{r_i \leftrightarrow r_j} B$, then $\delta(B) = -\delta(A)$.
(b) If $A \rightarrow_{r_i \rightarrow cr_i} B$, then $\delta(B) = c\delta(A)$.
(c) If $A \rightarrow_{r_i \rightarrow r_i + cr_j} B$, then $\delta(B) = \delta(A)$.

Prove that $\delta(A) = \det(A)$ for all $A \in \mathcal{M}_{2 \times 2}(F)$.

(10) (Due 11/21/17) Let $V$ be a finite dimensional vector space over $\mathbb{C}$ and let $T : V \to V$ be a linear transformation. Let $\lambda_1, \lambda_2, \ldots, \lambda_k$ be the distinct eigenvalues of $T$ and let $m_i$ be the algebraic multiplicity of $\lambda_i$. Prove that $\det(T) = \lambda_1^{m_1} \lambda_2^{m_2} \ldots \lambda_k^{m_k}$.

(11) (Due 12/5/17) Let $V$ be a vector space over either $\mathbb{R}$ or $\mathbb{C}$ and let $\beta$ be a basis of $V$. Prove that there exists an inner product on $V$ such that $\beta$ is an orthonormal basis.