OPTIMIZATION PROBLEMS

(1) A farmer has 2400 ft of fencing and want to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

(2) A box with a square base and an open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

(3) Find the coordinates of the point on the curve $y = x^2$ closest to the point $(2, \frac{1}{2})$.

(4) A cylindrical can is to be made to hold 1 L of liquid. Find the dimensions that will minimize the amount of material used to manufacture the can.

(5) An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 6 km east of the refinery. The pipe is going to be laid over land to a point P on the north bank and then under water in a straight line from P to the storage tanks. If laying pipe under water costs twice as much as laying pipe on land, where should P be located to minimize the cost of the pipe?

(6) The management of a local store has decided to enclose an 800 square foot area outside the building for a garden display. One side will be formed by an external wall of the store, 2 sides will be constructed of pineboards costing \$6 per foot and the side opposite the store will be constructed of fencing that costs \$3 per foot. What dimensions of the enclosure will minimize the cost?

(7) Find two nonnegative numbers so that the product of the first and the cube of the second is a maximum, if the sum of twice the first and three times the second is 60.

(8) A closed rectangular box is to be constructed with a surface area of 48 square feet so that its length is twice the width. What dimensions will maximize the volume of the box? What is the maximum volume?

(9) (a) Show that the profit $P(x)$ is maximized when marginal revenue equals marginal cost.
(b) If $C(x) = 16,000+500x-1.6x^2+.004x^3$ is the cost function and $p(x) = 1700 - 7x$ is the demand function, find the production level that will maximize profit.

(10) When fresh lemonade drinks sell for \$4.00 each at a college stadium concession stand, an average of 3000 will sell on a hot day. The concessions manager has observed that when he raises the price by 20 cents, an average of 300 fewer will sell during the game. Assuming the demand function is linear, if the manager has fixed costs of \$1250 per game and variable costs are 80 cents per unit, find the price of the drink that will maximize his profit.