MATH 155A-1 Test 1 Name:____________________

Show all work for full credit.
I have neither given nor received aid on this test. *Pledged:*____________________

Grade:______________________________

1. (4 points) Find the domain of $g(t) = \sqrt{6 + t - t^2}$

$6 + T - T^2 \geq 0$

$T^2 - T - 6 \geq 0$

$(T-3)(T+2) \geq 0$

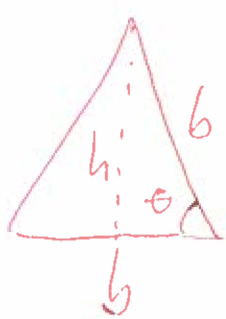
$-2 \quad 3$

$(T+2)$	$-$	$+$	$+$
$(T-3)$	$-$	$-$	$+$
$(T+2)(T-3)$	$+$	$-$	$+$

$(-\infty, -2], [3, \infty)$

2. (3 points) Complete the square: $x^2 + 6x + 7$

3. (8 points) Express the area of an equilateral triangle as a function of the length of one of the sides.



$A = \frac{1}{2} bh = \frac{1}{2} b\left(\frac{\sqrt{3}}{2} b\right)$

$\theta = \pi/3$

$\sin\theta = \frac{h}{b}$

$\frac{\sqrt{3}}{2} = \frac{h}{b}$

$h = \left(\sqrt{3}\right)b$

$A = \frac{\sqrt{3}}{4} b^2$

4. (8 points) Find all θ in $[0, 2\pi]$ such that $\sin(2\theta) = \cos(\theta)$

$2 \sin\theta \cos\theta = \cos\theta$

$2 \sin\theta \cos\theta - \cos\theta = 0$

$\cos\theta (2 \sin\theta - 1) = 0$

$\cos\theta = 0$ $\qquad$ $2 \sin\theta - 1 = 0$

$\sin\theta = 1/2$

$\theta = \pi/2, \ 3\pi/2$ $\qquad$ $\theta = \pi/6, \ 5\pi/6$

5. (6 points) Given $f(x) = \dfrac{1-x}{3x}$ and $g(x) = \dfrac{1}{1+3x}$, find and simplify $f \circ g$ and state its domain.

$$f(g(x)) = f\left(\frac{1}{1+3x}\right) = \frac{1 - \frac{1}{1+3x}}{3\left(\frac{1}{1+3x}\right)} \cdot \frac{1+3x}{1+3x}$$

$$= \frac{(1+3x) - 1}{3} = \frac{3x}{3} = x$$

domain: $x \neq -1/3$

6. (8 points) Find $\lim_{x \to 0} (\sqrt{x^4 + x^2}) \sin\left(\dfrac{\pi}{x}\right)$

$-1 \leq \sin\frac{\pi}{x} \leq 1$

$-\sqrt{x^4+x^2} \leq (\sqrt{x^4+x^2}) \sin\left(\frac{\pi}{x}\right) \leq \sqrt{x^4+x^2}$

$\lim_{x \to 0} \sqrt{x^4+x^2} = 0 = \lim_{x \to 0} -\sqrt{x^4+x^2}$

By Squeeze Theorem $\lim \sqrt{x^4+x^2} \sin\left(\frac{\pi}{x}\right) = 0$

7. (12 points) Evaluate the following limits.

(a) $\lim_{h \to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$ $= \lim_{h \to 0} \left(\frac{1}{h}\right)\left(\frac{1}{3+h} - \frac{1}{3}\right) = \lim_{h \to 0} \frac{1}{h}\left(\frac{3-(3+h)}{3(3+h)}\right)$

$= \lim_{h \to 0} \frac{1}{3(3+h)} = \frac{1}{9}$

(b) $\lim_{x \to 0} a$

$= a$

(c) $\lim_{x \to 2} \sqrt{\dfrac{2x^2+1}{3x-2}}$ $= \sqrt{\frac{8+1}{6-2}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

(d) $\lim_{x \to 0} \dfrac{x+3}{x^2}$ $= +\infty$

"$\frac{3}{0}$"

	-3		0	
$x+3$	$-$	$+$		$+$
x^2	$+$	$+$		$+$

(e) $\lim_{u \to 1} \dfrac{u^4-1}{u^3-1}$ $= \lim_{u \to 1} \frac{(u-1)(u^3+u^2+u+1)}{(u-1)(u^2+u+1)}$

$= \frac{4}{3}$

(f) $\lim_{x \to 0} \dfrac{1}{x} - \dfrac{1}{|x|}$ $=$ DNE

$\lim_{x \to 0^+} \frac{1}{x} - \frac{1}{x} = 0$

$\lim_{x \to 0^-} \frac{1}{x} + \frac{1}{x} = \lim_{x \to 0^-} \frac{2}{x} = -\infty$

8. **(10 points) Use the δ-ϵ definition of a limit to prove that** $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$

9. **(8 points) Let**

$$f(x) = \begin{cases} \cos(x) - 1 & x < 0 \\ 0 & x = 0 \\ x - x^2 & x > 0 \end{cases}$$

Explain why f is continuous on $(-\infty, \infty)$.

$\cos(x) - 1$ is continuous, so f is cont for $x < 0$.

at 0 $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \cos(x) - 1 = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x - x^2 = 0$

$f(0) = 0$, so f is cont at 0.

~~at~~ $x > 0$, $x - x^2$ is cont, so

f is cont. everywhere.

10. **(8 points)**

(a) **State the Intermediate Value Theorem.**

(b) **Show there exists a number in $[1, 2]$ that is exactly one less then its cube.**

11. **(10 points) Let $f(x) = \sqrt{1-2x}$.**

(a) **Find a formula for $f'(x)$, using the limit definition of the derivative.**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{1-2(x+h)} -}{h}$$

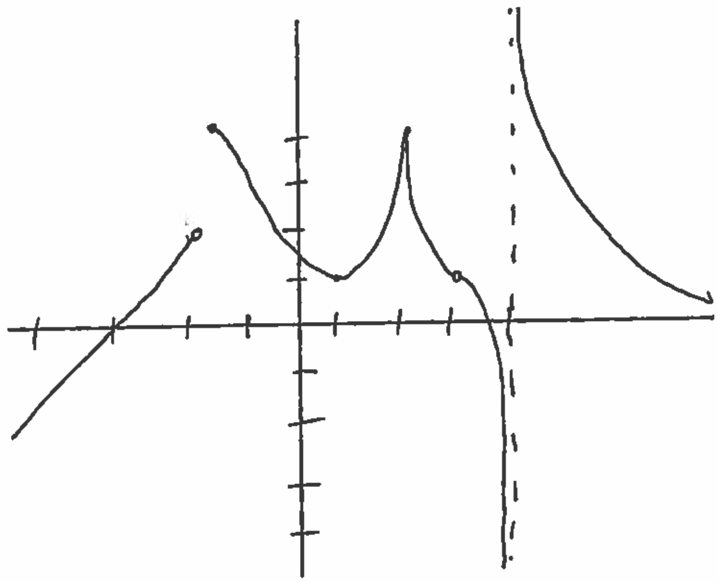
$$= \ldots = \tfrac{1}{2}(1-2x)^{-1/2}(-2) = -(1-2x)^{-1/2}$$

(b) **Give the equation of the line tangent to f at $(-4, 3)$.**

$$f'(-4) = -(1+8)^{-1/2} = -1/3$$

$$y - 3 = -\tfrac{1}{3}(x+4)$$

12. **(15 points) Let $f(x)$ be the function with the graph:**



(a) At which points is f *not* continuous? State which type of discontinuity occurs at each point.

$x=-2$ jump $\quad x=3$ removable $\quad x=4$ infinite

(b) At which points is f *not* differentiable?

$x=-2$, $x=3$, $x=4$, and $x=2$ (cusp)

(c) Sketch the graph of $f'(x)$