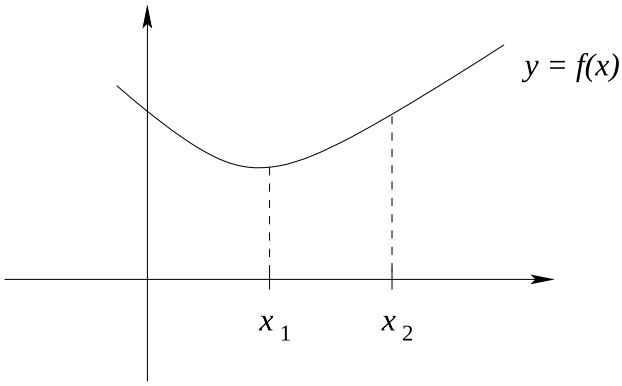
Pre-Lecture 18: Rates of Change (Section 3.7)

Recall the following:

Average Rate of Change



Instantaneous Rate of Change

<u>ex.</u> Let $Y(N)$ be the yield of an agricultural crop as a function of the nitrogen level N in the soil. One model is

$$Y(N) = \frac{100N}{1+N^2}, \; N \geq 0.$$

1) Find the average rate at which the yield is changing between $N = 1$ and $N = 3$.

2) For what nitrogen levels is the yield increasing? Decreasing? When is the yield maximized?

Lecture 18: Rates of Change

Physics

ex. Suppose the position of a particle is given by

$$s = f(t) = 2t^3 - 15t^2 + 24t,$$

where t is measured in seconds and s in feet.

1) Find the velocity of the particle at any time t.

2) Find the velocity at $t = 3$ seconds.

3) When is the particle at rest?

4) When is the particle moving in a positive direction?

5) Draw a diagram to represent the particle's motion.

6) Find the total distance the particle moves in the first six seconds.

Acceleration

7) Find the acceleration of $s(t) = 2t^3 - 15t^2 + 24t$ at any time t.

8) When is the particle **speeding up** and when is it **slowing down**?

Economics

The total cost of producing x units of a product is called the **cost function** $C(x)$.

Average rate of change of cost as the number of items produced increases from x_1 to x_2:

Marginal Cost

ex. Suppose the cost function for a certain product is given by $C(x) = 1000 + 25x - 0.1x^2$.

1) Find the total cost of producing 100 items.

2) Estimate the marginal cost at the production level of 100 items.

3) Find the actual cost of producing the 101st item given $C(101) = 1000 + 25(101) - 0.1(101)^2 = 2504.90$.

Now suppose that the unit price p at which x items will sell can be modeled by the **demand function**

$$p(x) = -0.3x + 125,\ 0 \leq x \leq 400.$$

4) Find the **revenue** from the sale of x items.

5) Find the profit function, $P(x)$, which gives the profit from the sale of x items.

6) Estimate the marginal profit when 50 items are sold.

Note: $P(51) - P(50) = 3579.80 - 3500$.

Chemistry

Consider a chemical reaction $A + B \to C$, where A and B are **reactants** and C is the **product**. The **concentration** of product C in moles per liter is denoted $[C](t)$.

Average rate of reaction of the product C over $[t_1, t_2]$ is

$$\frac{\Delta[C]}{\Delta t} = \frac{[C](t_2) - [C](t_1)}{t_2 - t_1}$$

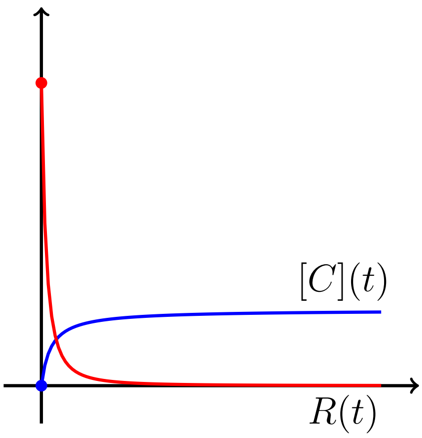
$$\text{Instantaneous rate of reaction} = \lim_{\Delta t \to 0} \frac{\Delta[C]}{\Delta t} = \frac{d[C]}{dt}$$

ex. Assume that the initial concentration of A and B have the same value $[A] = [B] = 2$ moles/L, and $[C](t) = \dfrac{16t}{8t+1}$ moles/L.

1) Find the rate of reaction at time t, $R(t)$.

2) Find the rate of reaction at $t = 0$, 1, and 2 seconds. Include units.

3) What happens to $[C](t)$ and $R(t)$ at $t \to \infty$. Does this make sense?



Biology

ex. A certain type of bacteria doubles in population every 3 hours. Assume that there were 10 bacteria initially.

1) Find a formula for the population of bacteria at any time t.

2) Find the rate of growth of the bacteria at $t = 6$ hour.