

MAP2302
Elementary Differential Equations
Exam 3

Name: _____

UFID: _____

Instructions:

- Read each problem carefully.
- Show all your work; you will not get credit for answers with no work even if they are correct.
- The proctor will not answer questions about the material on the exam or give hints to any of the problems; do your best to answer each question as written.
- Students should not have calculators, phones, or paper on their desk and they should not wear headphones. No student writing should be in a position where it is visible.
- All numerical answers should be left in exact form (i.e. use $\ln(2)$, not $\approx .7$).
- The proctor will have additional scratch paper if needed.

1. Three identical springs with spring constant $k = 3$ N/m and two objects of identical mass $m = 1$ kg are attached in a straight line with the ends of the outside springs fixed. If $x(t)$ denotes the position of the first object at time t and $y(t)$ denotes the position of the second object, then this system is governed by the equations

$$mx'' = -kx + k(y - x)$$

$$my'' = -k(y - x) - ky.$$

Suppose $x(0) = 6$, $y(0) = 2$, and $x'(0) = y'(0) = 0$. Solve for $x(t)$ and $y(t)$.

2. Give a fundamental solution set for the homogeneous equation

$$(D^2 - 2D + 3)^3(D^4 + 2D^3 - 15D^2)[y] = 0.$$

3. Find a differential operator A such that A annihilates $e^x + 3e^{2x}$. Then determine a particular solution to

$$y''' - y' = e^x + 3e^{2x}.$$

4. Given that $\{1, x^2, x^{-2}\}$ is a fundamental solution set for $y''' + 3x^{-1}y'' - 3x^{-2}y' = 0$ on the interval $x > 0$, determine a particular solution on this interval to

$$y''' + 3x^{-1}y'' - 3x^{-2}y' = x^{-1}.$$