MATH 155A-1   Test 1   Name:_____

Show all work for full credit.
I have neither given nor received aid on this test. *Pledged:*_____

Grade:_____

1. (4 points) Find the domain of $g(t) = \sqrt{6 + t - t^2}$

2. (3 points) Complete the square: $x^2 + 6x + 7$

3. (8 points) Express the area of an equilateral triangle as a function of the length of one of the sides.

4. (8 points) Find all θ in $[0, 2\pi]$ such that $\sin(2\theta) = \cos(\theta)$

5. (6 points) Given $f(x) = \dfrac{1-x}{3x}$ and $g(x) = \dfrac{1}{1+3x}$, find and simplify $f \circ g$ and state its domain.

6. (8 points) Find $\lim\limits_{x \to 0}(\sqrt{x^4 + x^2})\sin\left(\dfrac{\pi}{x}\right)$

7. (12 points) Evaluate the following limits.

(a) $\lim\limits_{h \to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$

(b) $\lim\limits_{x \to 0} a$

(c) $\lim\limits_{x \to 2} \sqrt{\dfrac{2x^2 + 1}{3x - 2}}$

(d) $\lim\limits_{x \to 0} \dfrac{x + 3}{x^2}$

(e) $\lim\limits_{u \to 1} \dfrac{u^4 - 1}{u^3 - 1}$

(f) $\lim\limits_{x \to 0} \dfrac{1}{x} - \dfrac{1}{|x|}$

8. (10 points) Use the δ-ϵ definition of a limit to prove that $\displaystyle\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$

9. (8 points) Let
$$f(x) = \begin{cases} \cos(x) - 1 & x < 0 \\ 0 & x = 0 \\ x - x^2 & x > 0 \end{cases}$$

Explain why f is continuous on $(-\infty, \infty)$.

10. (8 points)

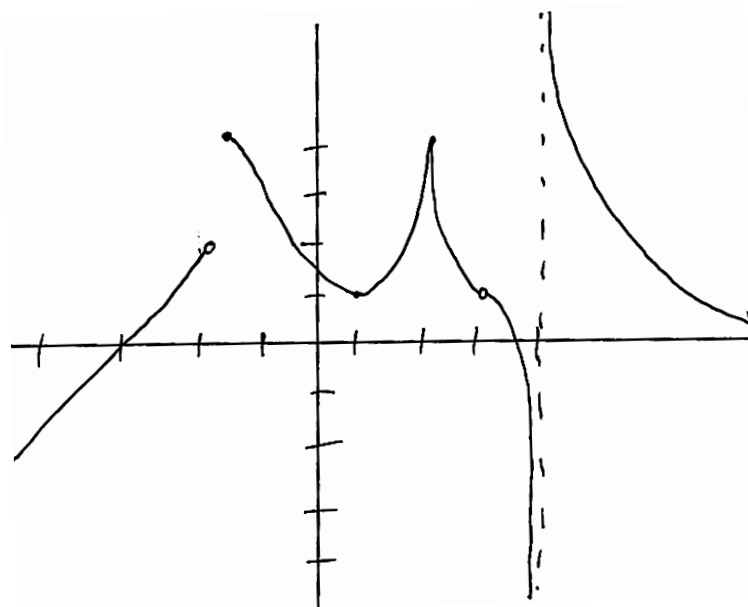   (a) State the Intermediate Value Theorem.

    (b) Show there exists a number in $[1, 2]$ that is exactly one less then its cube.

11. (10 points) Let $f(x) = \sqrt{1 - 2x}$.

    (a) Find a formula for $f'(x)$, using the limit definition of the derivative.

    (b) Give the equation of the line tangent to f at $(-4, 3)$.

12. (15 points) Let $f(x)$ be the function with the graph:

(a) **At which points is f *not* continuous? State which type of discontinuity occurs at each point.**

(b) **At which points is f *not* differentiable?**

(c) **Sketch the graph of $f'(x)$**