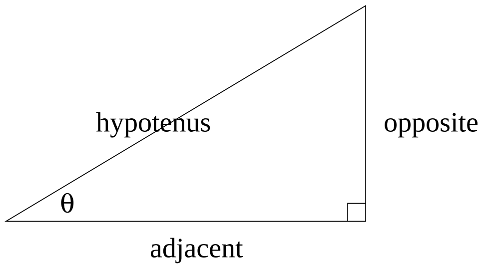
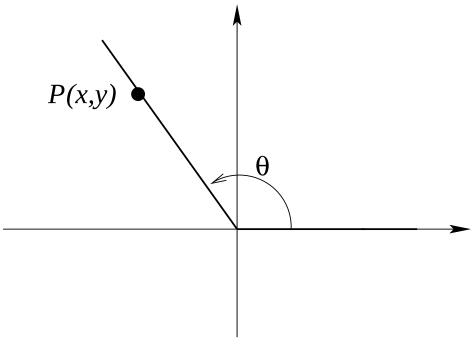
Pre-Lecture 3: Precalculus Trigonometry (Sections 1.2, 1.3 & 1.5)

Trigonometric Functions

1. Trigonometric functions





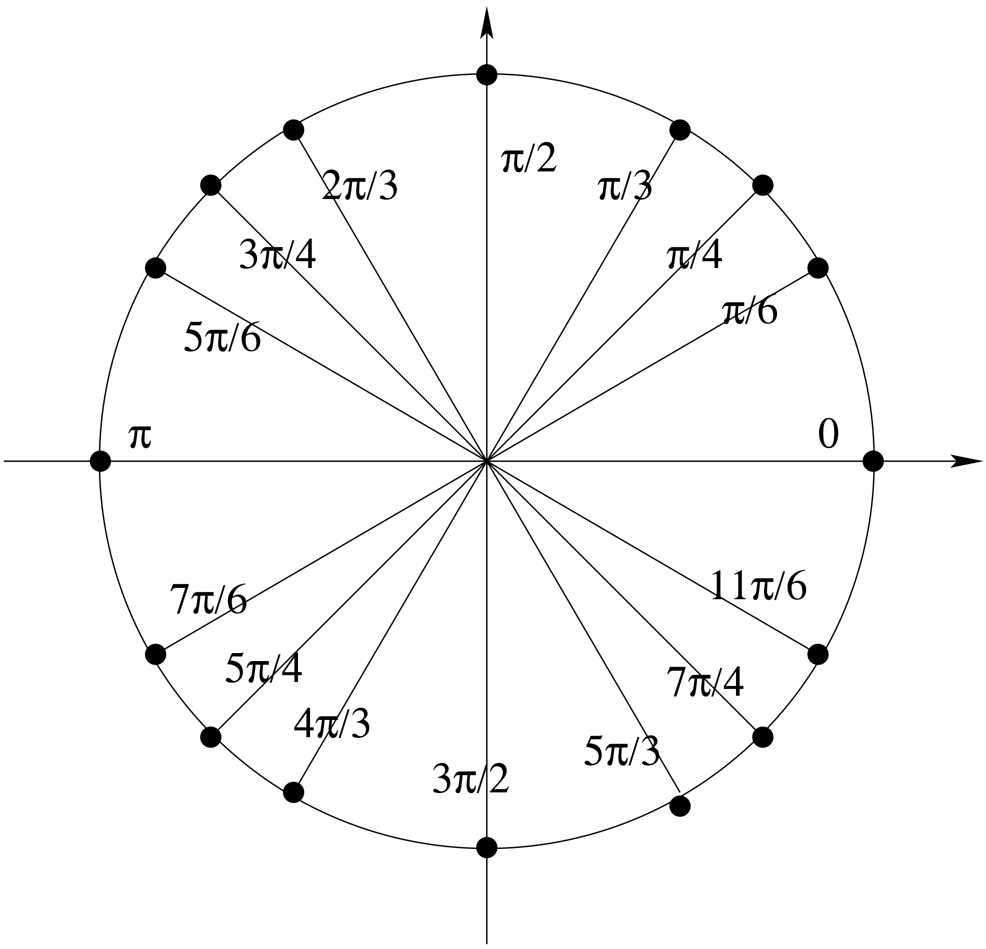
$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

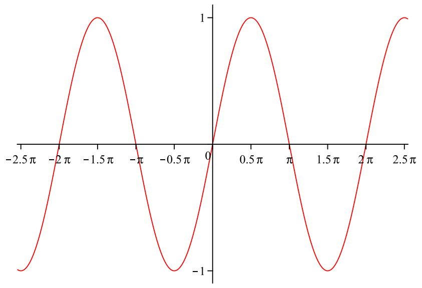
In general, let $P(x, y)$ be any point on the terminal side of θ (radians) and let $r = \sqrt{x^2 + y^2}$ be the distance from the origin to point P.

2. Unit circle ($r = 1$, so $\sin\theta = y$ and $\cos\theta = x$)

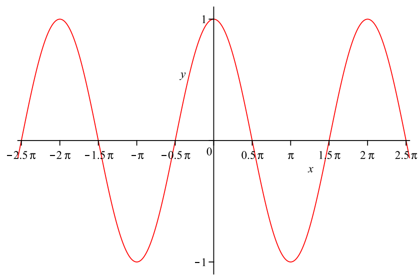


θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					

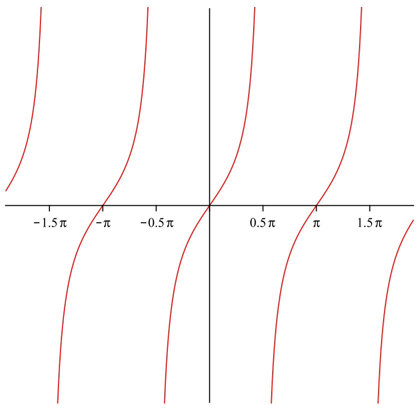
3. Graphs



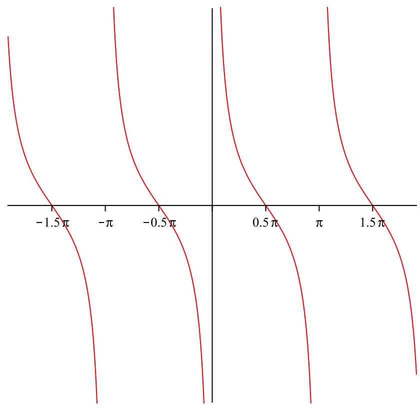
$y = \sin x$



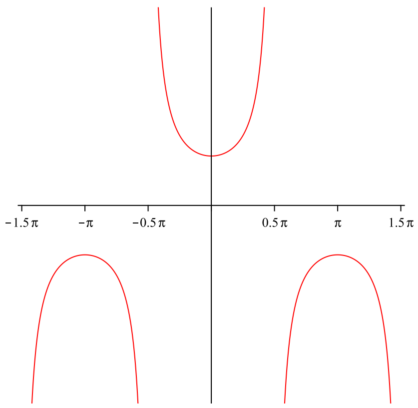
$y = \cos x$



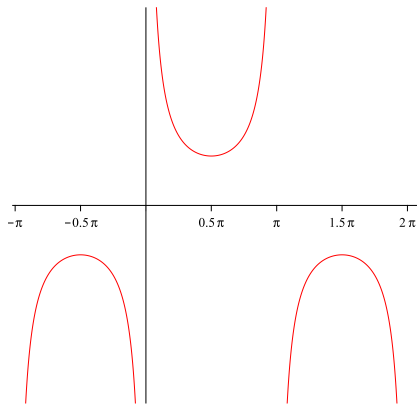
$y = \tan x$



$y = \cot x$



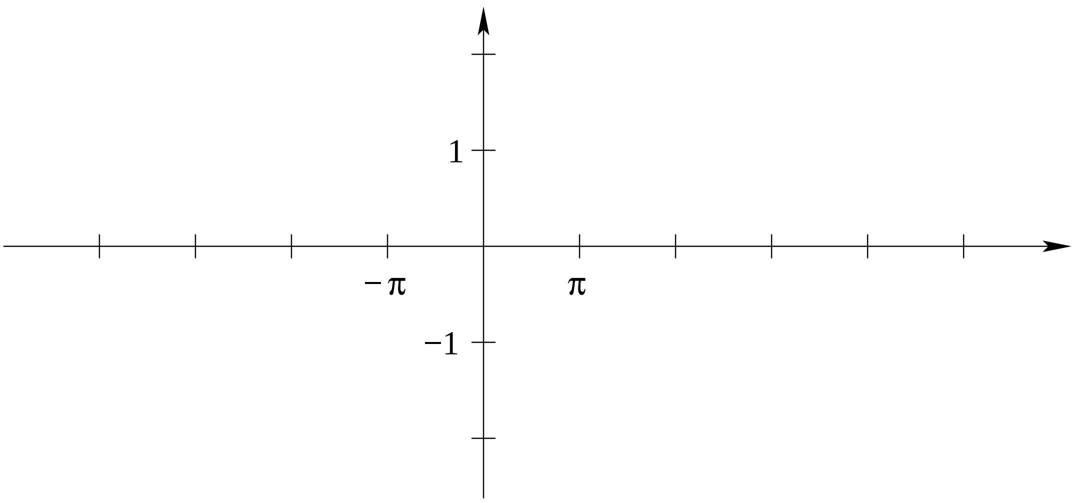
$y = \sec x$



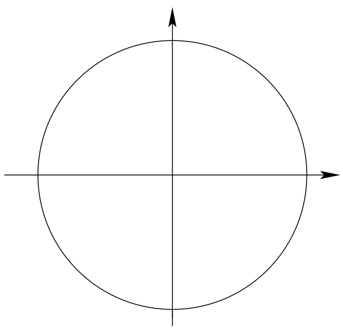
$y = \csc x$

NOTE: $\quad \leq \sin x \leq \quad , \quad \leq \cos x \leq$

ex. Sketch the graph of $y = -2\sin\left(\frac{1}{2}x\right)$.



Basic Trigonometric Identities



1) $\sin^2\theta + \cos^2\theta = 1$

2) $\tan^2\theta + 1 = \sec^2\theta$

3) $1 + \cot^2\theta = \csc^2\theta$

4) $\sin(-\theta) = -\sin\theta$

5) $\cos(-\theta) = \cos\theta$

Addition/Subtraction Formulas

6) $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

7) $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

8) $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

Double-Angle Formulas

9) $\sin(2x) = 2 \sin x \cos x$

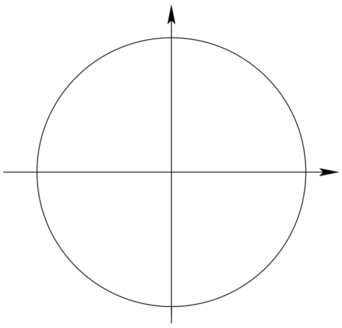
10) $$\begin{aligned} \cos(2x) &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

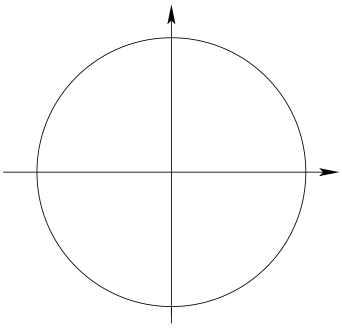
Half-Angle Formulas

11) $\cos^2 x = \dfrac{1 + \cos(2x)}{2}$

12) $\sin^2 x = \dfrac{1 - \cos(2x)}{2}$

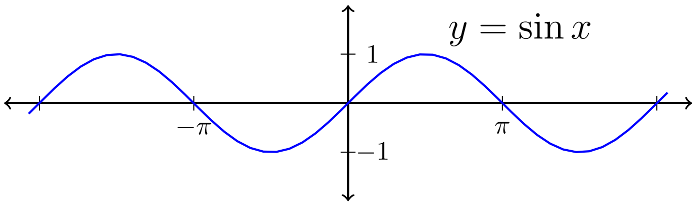
<u>**ex.**</u> If α is in the third quadrant with $\tan\alpha = \frac{2}{3}$ and β is in the first quadrant with $\sin\beta = \frac{3}{5}$, find the exact value of $\sec(\alpha + \beta)$.

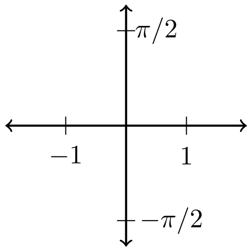




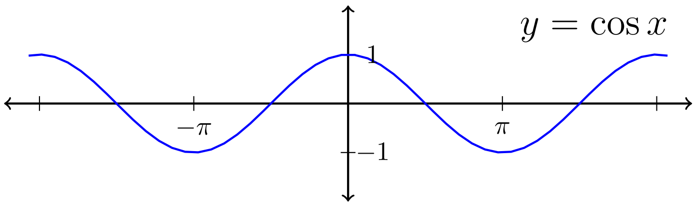
Inverse Trigonometric Functions

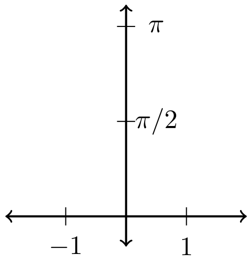
- $y = \sin^{-1} x$ if and only if



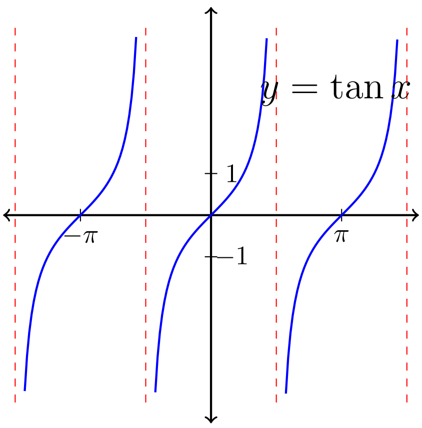


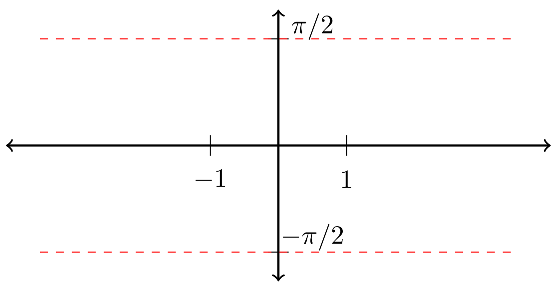
- $y = \cos^{-1} x$ if and only if





- $y = \tan^{-1} x$ if and only if

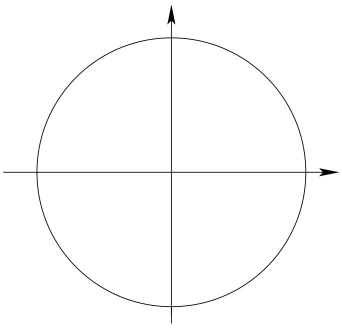




There are similar definitions for the inverse of the other trigonometric functions.

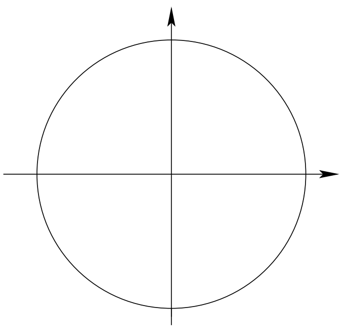
ex. Find the following if possible:

1) $\sin^{-1}\left(-\frac{1}{2}\right)$



2) $\cos^{-1}(2)$

3) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$



Lecture 3: Precalculus Trigonometry

Inverse Properties:

1. $\sin(\sin^{-1} x) = x \quad$ for $-1 \leq x \leq 1$

   $\sin^{-1}(\sin x) = x \quad$ for $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$

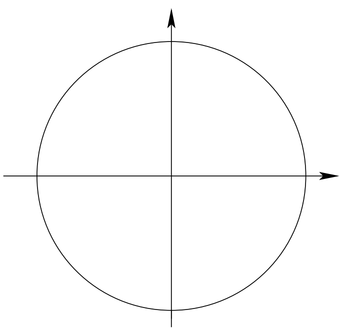
2. $\cos(\cos^{-1} x) = x \quad$ for $-1 \leq x \leq 1$

   $\cos^{-1}(\cos x) = x \quad$ for $0 \leq x \leq \pi$

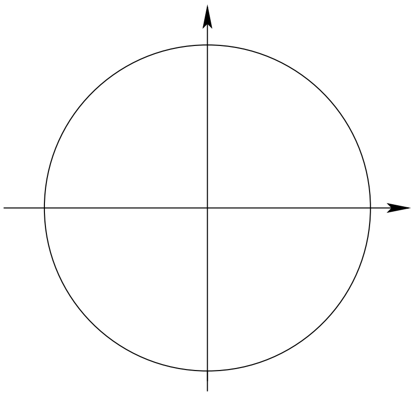
3. $\tan(\tan^{-1} x) = x \quad$ for all x

   $\tan^{-1}(\tan x) = x \quad$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

<u>ex.</u> $\tan^{-1}\left(\tan \dfrac{7\pi}{5}\right) =$



ex. Use a triangle to find the exact value: $\sin(\tan^{-1}(-2))$



ex. Use a triangle to simplify the expression: $\cos(2\tan^{-1} x)$

Trig Equation

ex. Solve for θ in $[0, 2\pi)$ if $\sqrt{3}\sin 2\theta + 2\sin^2\theta = 0$.

Trig Inequality

ex. Solve for θ in $[0, 2\pi)$ where $\sin\theta > \tan\theta$.

Now You Try It (NYTI):

1. Is $f(x) = \dfrac{x^2 - \cos x}{x}$ even, odd, or neither? Verify your answer. odd

2. Suppose that $\sin\alpha = -\dfrac{3}{5}$, $\cot\beta = -\dfrac{2}{3}$, $\pi < \alpha < \dfrac{3\pi}{2}$ and $\dfrac{3\pi}{2} < \beta < 2\pi$. Find:
   (a) $\cos(2\alpha)$ $\qquad \dfrac{7}{25}$
   (b) $\sin(\alpha + \beta)$ $\qquad \dfrac{6}{5\sqrt{13}}$

3. Solve each equation for θ on the interval $[0, 2\pi)$:
   (a) $\sin(2\theta) = \sqrt{2}\sin(\theta)$ $\qquad 0,\ \pi/4,\ \pi,\ 7\pi/4$
   (b) $2\cos^2(\theta) - \sin(\theta) = 1$ $\qquad \pi/6,\ 5\pi/6,\ 3\pi/2$
   (c) $\sec^2(\theta) - 2\tan(\theta) = 0$ $\qquad \pi/4,\ 5\pi/4$
   (d) $2\sin^4(\theta) - 9\sin^2(\theta) + 4 = 0$ $\qquad \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$

4. Find all values of x on the interval $[0, 2\pi)$ that satisfy each inequality:
   (a) $2\sin(x) \leq \sqrt{3}$ $\qquad \left[0, \dfrac{\pi}{3}\right] \cup \left[\dfrac{2\pi}{3}, 2\pi\right)$
   (b) $\cos(x) > \sin(x)$ $\qquad \left[0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{5\pi}{4}, 2\pi\right)$

5. Use a triangle to simplify $\csc\left(\cos^{-1}\left(\dfrac{x}{2}\right)\right)$. $\qquad \dfrac{2}{\sqrt{4 - x^2}}$