1. Find the domain, x intercept and vertical asymptote of the function

$$f(x) = \log_5(x-4) \tag{1}$$

Solution: The function log is defined only for positive quantities. Therefore,

$$x-4>0 \qquad \longmapsto x>4 \tag{2}$$

The x intercept of log function is where it equals to 1. Hence

$$x-4=1 \qquad \longmapsto x=5 \tag{3}$$

log function has asymptote where it equals zero. As the result,

$$x-4=0 \qquad \longmapsto x=4 \tag{4}$$

2. Use the properties of logarithms to expand the expression as sum, difference or constant multiple of logarithm.

$$\log(\frac{x^2-3x+2}{\sqrt[3]{x^5}}) \tag{5}$$

Solution :

$$\log(\frac{x^2-3x+2}{\sqrt[3]{x^5}}) \quad = \quad \log(x^2-3x+2) - \log(\sqrt[3]{x^5}) \tag{6}$$

$$= \log((x-1)(x-2)) - \log(x^{\frac{5}{3}}) \quad = \quad \log(x-1) + \log(x-2) - \frac{5}{3}\log(x) \tag{7}$$

3. Solve the logarithm equation.

$$\log_4(2x+1) - \log_4(2x-1) = \frac{1}{2} \tag{8}$$

Solution :

$$\log_4(2x+1) - \log_4(2x-1) = \frac{1}{2} \quad = \quad \log_4(\frac{2x+1}{2x-1}) \quad = \quad \frac{1}{2} \tag{9}$$

Now, by using the definition of logarithm, one can rewrite above equation as

$$\frac{2x+1}{2x-1} = 4^{\frac{1}{2}} \quad = \quad 2 \tag{10}$$

$$2x+1 = 2(2x-1) \quad = \quad 4x-2 \longmapsto x = -\frac{3}{2} \tag{11}$$

However, $-\frac{3}{2}$ does not belong to domain of $\log(2x-1)$. Since, log is not defined for negative functions. Please be careful! Always check your answer to see if the are acceptable.

4. If $\log(x+2) = 3$ and $\log(x-2) = 4$, find the value of

$$f(x) = x^2 - 4 \tag{12}$$

Solution : First, one can calculate $\log(f(x))$

$$\log(f(x)) = \log(x^2-4) \quad = \quad \log((x-2)(x+2) \quad = \quad \log(x-2)+\log(x+2) \quad = \quad 3+4 \quad = \quad 7 \tag{13}$$

Therefore,by using the definition of logarithm

$$\log(f(x)) = 7 \qquad \longmapsto f(x) = 10^7 \tag{14}$$