Solution 1: D is correct! By using the definition of tangent we have
\[ \tan(\theta) = \frac{\text{opp}}{\text{adj}} \] (1)
On the other hand
\[ \tan\left(\frac{\pi}{6}\right) = \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3} \] (2)
By equating equations 1 and 2 and the fact that adj = 1, one has
\[ \frac{\text{opp}}{1} = \frac{\sqrt{3}}{3} \implies \text{opp} = \frac{\sqrt{3}}{3} \] (3)
Solution 2: B is correct! By considering the definition of sec function we know that
\[ \sec\left(\frac{\pi x}{2}\right) = \frac{1}{\cos\left(\frac{\pi x}{2}\right)} \] (4)
Notice that for \( x = 1 \) and \( x = 3 \) the value of \( \cos\left(\frac{\pi x}{2}\right) \) is zero making denominator zero. Hence, both \( x = 1 \) and \( x = 3 \) are vertical asymptotes of the function \( \sec\left(\frac{\pi x}{2}\right) \). In addition, by recalling the graph of this function, one can realize that the range varies between \((-\infty, \frac{1}{2})\) and \(\left(\frac{1}{2}, \infty\right)\), therefore all statements are true.
Solution 3: D is correct! recall the fact that
\[ \cos(\theta) = \sin(90^\circ - \theta) \] (5)
Therefore, one can replace \( \cos(70^\circ) \) with \( \sin(90^\circ - 70^\circ) = \sin(20^\circ) \) and rewrite the equation as
\[ \cos(20^\circ)^2 + \cos(70^\circ)^2 = \cos(20^\circ)^2 + \sin(20^\circ)^2 \] (6)
Since, for any \( \theta \) the identity
\[ \cos(\theta)^2 + \sin(\theta)^2 = 1 \] (7)
is true we have
\[ \cos(20^\circ)^2 + \sin(20^\circ)^2 = 1 \] (8)
Solution 4: A is correct! First, recall the definition of \( \cot(2x) \).
\[ \cot(2x) = \frac{\cos(2x)}{\sin(2x)} \] (9)
For \( x = 0 \) and \( x = \pi \) the value of denominator is zero determining that \( \cot(2x) \) has above lines as its vertical asymptotes. On the other hand \( \cot(2x) \) is an decreasing function in
one of its complete periods meaning that as \( x \) increases in this interval the value of \( \cot(2x) \) decreases. By considering above facts, the only possible solution is choice A.

Solution 5 : D is correct! First, notice that the function \( y = a \sin(bx - c) \) passes the origin which means that \( y(0) = 0 \). Hence,

\[
a \sin(b(0) - c) = 0 \quad \iff \quad a \sin(-c) = -a \sin(c) = 0 \quad \iff \quad c = 0
\]  

(10)

On the other hand, we know that after zero the nearest root of \( \sin \) function in direction of positive numbers is \( x = \pi \). But, the function \( y = a \sin(bx) \) intersects \( x \)-axis in the direction of positive numbers at \( x = \frac{\mu}{2} \) for the first time as it is shown in our graph meaning that \( y(\frac{\pi}{2}) = 0 \). Therefore,

\[
a \sin(b(\frac{\pi}{2})) = 0 \quad \iff \quad b(\frac{\pi}{2}) = \pi \quad \iff \quad b = 2
\]  

(11)

Solution 6 : D is correct! By using the relation

\[
\arccos(x) = \arcsin(\sqrt{1-x^2})
\]  

(12)

one can solve this problem as follow,

\[
\arccos(\frac{4}{\sqrt{x^2 + 4x + 20}}) = \arcsin(\sqrt{1 - (\frac{4}{\sqrt{x^2 + 4x + 20}})^2})
\]  

(13)

and by simplifying the expression under radical sign one has

\[
\sqrt{1 - (\frac{4}{\sqrt{x^2 + 4x + 20}})^2} = \sqrt{\frac{x^2 + 4x + 20 - 16}{x^2 + 4x + 20}} = \sqrt{\frac{x^2 + 4x + 4}{x^2 + 4x + 20}}
\]  

(14)

One can go further and rewrite \( x^2 + 4x + 4 \) as

\[
(x + 2)^2
\]  

(15)

Therefore, by replacing 14 and 15 in 12, final answer would be

\[
\arccos(\frac{4}{\sqrt{x^2 + 4x + 20}}) = \arcsin(\frac{|x + 2|}{\sqrt{x^2 + 4x + 20}})
\]  

(16)

Solution 7 : D is correct! As we know the domain of the function \( \arcsin \) is only defined on the interval \( \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] \). Both \( x = \frac{-\pi}{4} \) and \( x = \frac{7\pi}{4} \) satisfy the equation

\[
\arcsin x = \frac{-\sqrt{2}}{2}
\]  

(17)
However, $x = \frac{7\pi}{4}$ does not belong to domain of arcsin. Therefore it is not acceptable.

Solution 8: D is correct! For every $t$ one has the identity
\[ \sin(-t) = -\sin(t) \] (18)
Hence, $\sin(-t) = -\sin(t) = -\frac{1}{5}.$

Solution 9: D is correct! Choose the point $A = (1,0)$ on x-axis. By replacing $x = 1$ in the given line $3x+y = 0$, one has
\[ 3 + y = 0 \quad \iff y = -3 \] (19)
Choose the point $B = (1,-3)$ on the given line. In this way one can consider $AOB$ as a right triangle in which adjacent $= 1$ and opposite $= -3$. Then, by using Pythagoras relation one can find the length of hypotenuse according to
\[ h^{2} = a^{2} + o^{2} = 1^{2} + (-3)^{2} = 10 \quad \iff h = \sqrt{10} \] (20)
Therefore, the value of $\sin(\theta)$ is equal to

\[ \sin(\theta) = \frac{o}{h} = \frac{-3}{\sqrt{10}} \quad \text{or} \quad = \frac{-3\sqrt{10}}{10} \] (21)

Solution 10: D is correct! Suppose $y_{1}$ and $y_{2}$ are the length of flagpole under and above our eye sight, respectively. In order to find value of $x$ one can use the definition of tangent function as follow,
\[ \tan(50^{\circ}) = \frac{y_{2}}{x} \quad \iff y_{2} = \tan(50^{\circ})x \] (22)
and
\[ \tan(15^{\circ}) = \frac{y_{1}}{x} \quad \iff y_{1} = \tan(15^{\circ})x \] (23)
We know that the length of flagpole is 80 meters which means that
\[ y_{1} + y_{2} = 80 \] (24)
By replacing 22 and 23 in 24 we have
\[ 80 = y_{1} + y_{2} = \tan(15^{\circ})x + \tan(50^{\circ})x = (\tan(50^{\circ}) + \tan(15^{\circ}))x \] (25)
Hence,
\[ x = \frac{80}{\tan(50^{\circ}) + \tan(15^{\circ})} \] (26)

Solution 11: C is correct! In this problem opp = 1500 and hyp = 3000. Therefore,
\[ \sin(\theta) = \frac{1500}{3000} = \frac{1}{2} \quad \iff \theta = \arcsin\left(\frac{1}{2}\right) = 30^{\circ} \] (27)
Solution 12: A is correct! We know that in isosceles triangle altitude divides $b$ into two sections with equal lengths. In this case, one can use the relation
\[ \tan(\theta) = \frac{\text{altitude}}{\text{half of } b} = \frac{alt}{10} \quad (28) \]

On the other hand, in an isosceles triangle all angles are equal $60^\circ$. Therefore, we have
\[ \tan(60^\circ) = \frac{\sin(60^\circ)}{\cos(60^\circ)} = \frac{\sqrt{3}}{\frac{1}{2}} = \sqrt{3} \quad (29) \]

By equating relations 28 and 29 one have
\[ \sqrt{3} = \frac{alt}{10} \implies alt = 10\sqrt{3} \quad (30) \]

Solution 13: B is correct! Recall that a function is odd if and only if $f(-x) = -(f(x))$. Here, if one replaces $-x$ with $x$ in $f(x) = x \cos(x)$, then
\[ f(-x) = (-x) \cos(-x) \quad (31) \]

Since, $\cos(-x) = \cos(x)$,
\[ f(-x) = (-x) \cos(-x) = -x \cos(x) \implies f(-x) = -f(x) \quad (32) \]

Therefore, $f(x)$ is odd. By recalling the graph of $\sin$ function one realizes that its value alternates between -1 and 1. Hence, Q is also correct. The $y = \tan(x)$ has two consecutive vertical axis in $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. Hence, the period of this function is $\pi$.

Solution 14: C is correct! From elementary physics course we know that if an object moves by constant velocity then it satisfies the relation
\[ x = vt \quad (33) \]

where, $x$ is distance, $v$ is for velocity and $t$ is time. According to this problem, this ship travels to north east for 2 hours. Therefore, its distance from port is
\[ x = vt \implies x = 10 \times 2 = 20 \quad (34) \]

and after that it ravels to the east for one hour with the same velocity. If we call this length $y$, then
\[ y = vt \implies y = 10 \times 1 = 10 \quad (35) \]

Now, one can connect origin to the point our ship started to travel east and north axis. It gives us a right triangle. In this case, opposite side is the distance from the ship and north axis and $x$ is hypotenuse of this triangle. Hence,
\[ \text{the distance from north axis} = 20 \sin(30^\circ) = 10 \quad (36) \]
The length of adjacent of this triangle is

\[ \text{adj} = 20 \cos(30^\circ) = 10\sqrt{3} \quad (37) \]

At 3 pm this ship has the distance \( \text{opp} + y = 20 \) from north axis. Therefore, one can use Pythagorean theorem in order to find the distance of our ship from port.

\[ \text{distance}^2 = 20^2 + 10\sqrt{3}^2 = 700 \quad \longrightarrow \text{dis} = 10\sqrt{7} \quad (38) \]

Solution 15 : E is correct!

\[ \cos\left(\frac{-19\pi}{6}\right) = \cos\left(\frac{19\pi}{6}\right) = \cos\left(\frac{18\pi + \pi}{6}\right) = \cos\left(3\pi + \frac{\pi}{6}\right) \quad (39) \]

Since \( \cos \) is a periodic function with period \( 2\pi \), one can ignore any multiplication of \( 2\pi \). Hence, in this case we have

\[ \cos\left(3\pi + \frac{\pi}{6}\right) = \cos\left(\pi + \frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2} \quad (40) \]

Solution 16 : C is correct! Recall that

\[ 1 + \tan(x)^2 = \frac{1}{\cos(x)^2} \quad (41) \]

Therefore, one can rewrite our expression in the form

\[ \ln(1 + \tan(x)^2) + \ln(\cos(x)^2) = \ln\left(\frac{1}{\cos(x)^2}\right) + \ln(\cos(x)^2) \quad (42) \]

Now, in order to simplify one can use the fact that

\[ \ln(a) + \ln(b) = \ln(ab) \quad (43) \]

In this case, we have

\[ \ln\left(\frac{1}{\cos(x)^2}\right) + \ln(\cos(x)^2) = \ln\left(\frac{\cos(x)^2}{\cos(x)^2}\right) = \ln(1) = 0 \quad (44) \]

Solution 17 : B is correct!

\[ \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{8}{5} \quad (45) \]

Therefore,

\[ \sin(\theta) = \frac{5}{8} \quad (46) \]
We know that \( \sin(\theta) \) has two positive value in quarter 1 and quarter 2. In order to find the unique value of \( \theta \) we must use other piece of information this problem gives us! Here, we know that \( \tan(\theta) \) is negative. But, all of basic trigonometric functions are positive at first quarter. Therefore, \( \theta \) belongs to second quarter. In order to find the value of \( \cos(\theta) \) one can use the relation

\[
\cos(\theta) = \sqrt{1 - \sin(\theta)^2}
\]  

(47)

In this case, we have

\[
\cos(\theta) = \sqrt{1 - \frac{25}{64}} = \sqrt{\frac{39}{64}}
\]

(48)

But, since \( \theta \) belongs to second quarter, the value of \( \cos(\theta) \) is negative. Hence

\[
\cos(\theta) = -\sqrt{\frac{39}{64}} \quad \text{or} \quad -\frac{\sqrt{39}}{8}
\]

(49)

Solution 18 : E is correct! One can use the relation

\[
\cos\left(\frac{\pi}{2} - x\right) = \sin(x)
\]

(50)

in order to simplify the given relation. Hence,

\[
\sin(x) = \frac{3}{5}
\]

(51)

Since \( \sin(x) > 0 \), there are two possible solution for above equation in first and second quarter. Since all simple trigonometric functions are positive in first quarter and by using the assumption that \( \cos(x) < 0 \), one can realize that \( x \) belongs to second quarter and its value is

\[
\cos(x) = -\sqrt{1 - \sin(x)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = -\frac{4}{5}
\]

(52)

Hence, \( \cot(x) \) equals to

\[
\cot x = \frac{\cos(x)}{\sin(x)} = \frac{-\frac{4}{5}}{\frac{3}{5}} = -\frac{4}{3}
\]

(53)