Solution 1 : D is correct! By using the definition of tangent we have

$$\tan(\theta) = \frac{opp}{adj} \tag{1}$$

On the other hand

$$\tan(\frac{\pi}{6}) \quad = \quad \frac{\sin(\frac{\pi}{6})}{\cos(\frac{\pi}{6})} \quad = \quad \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \quad = \quad \frac{1}{\sqrt{3}} \quad or \quad \frac{\sqrt{3}}{3} \tag{2}$$

By equating equations 1 and 2 and the fact that $adj = 1$, one has

$$\frac{opp}{1} \quad = \quad \frac{\sqrt{3}}{3} \quad \longmapsto \quad opp \quad = \quad \frac{\sqrt{3}}{3} \tag{3}$$

Solution 2 : B is correct! By considering the definition of sec function we know that

$$\sec(\frac{\pi x}{2}) = \frac{1}{\cos(\frac{\pi x}{2})} \tag{4}$$

Notice that for $x = 1 \quad and \quad x = 3$ the value of $\cos(\frac{(\pi x)}{2})$ is zero making denominator zero. Hence, both $x = 1 \quad and \quad x = 3$ are vertical asymptotes of the function $\sec(\frac{\pi x}{2})$. In addition, by recalling the graph of this function, one can realize that the range varies between $(-\infty, \frac{-1}{2})$ and $(\frac{1}{2}, \infty)$, therefore all statements are true.

Solution 3 : D is correct! recall the fact that

$$\cos(\theta) \quad = \quad \sin(90° - \theta) \tag{5}$$

Therefore, one can replace $\cos(70°)$ with $\sin(90° - 70°) = \sin(20°)$ and rewrite the equation as

$$\cos(20°)^2 + \cos(70°)^2 \quad = \quad \cos(20°)^2 + \sin(20°)^2 \tag{6}$$

Since, for any θ the identity

$$\cos(\theta)^2 + \sin(\theta)^2 = 1 \tag{7}$$

is true we have

$$\cos(20°)^2 + \sin(20°)^2 = 1 \tag{8}$$

Solution 4 : A is correct! First, recall the definition of $\cot(2x)$.

$$\cot(2x) \quad = \quad \frac{\cos(2x)}{\sin(2x)} \tag{9}$$

For $x = 0$ and $x = \pi$ the value of denominator is zero determining that $\cot(2x)$ has above lines as its vertical asymptotes. On the other hand $\cot(2x)$ is an decreasing function in one of its complete periods meaning that as x increases in this interval the value of $\cot(2x)$ decreases. By considering above facts, the only possible solution is choice A.

Solution 5 : D is correct! First, notice that the function $y = a\sin(bx - c)$ passes the origin which means that $y(0) = 0$.Hence,

$$a\sin(b(0) - c) = 0 \qquad \longmapsto a\sin(-c) = -a\sin(c) = 0 \qquad \longmapsto c = 0 \tag{10}$$

On the other hand,we know that after zero the nearest root of sin function in direction of positive numbers is $x = \pi$. But, the function $y = a\sin(bx)$ intersects x-axis in the direction of positive numbers at $x = \frac{pi}{2}$ for the first time as it is shown in our graph meaning that $y(\frac{\pi}{2}) = 0$. Therefore,

$$a\sin(b(\frac{\pi}{2})) = 0 \qquad \longmapsto b(\frac{\pi}{2}) = \pi \qquad \longmapsto b = 2 \tag{11}$$

Solution 6 : D is correct! By using the relation

$$\arccos(x) = \arcsin(\sqrt{1 - x^2}) \tag{12}$$

one can solve this problem as follow,

$$\arccos(\frac{4}{\sqrt{x^2 + 4x + 20}}) = \arcsin(\sqrt{1 - (\frac{4}{\sqrt{x^2 + 4x + 20}}^2)}) \tag{13}$$

and by simplifying the expression under radical sign one has

$$\sqrt{1 - (\frac{4}{\sqrt{x^2 + 4x + 20}}^2)} = \sqrt{\frac{x^2 + 4x + 20 - 16}{x^2 + 4x + 20}} = \sqrt{\frac{x^2 + 4x + 4}{x^2 + 4x + 20}} \tag{14}$$

One can go further and rewrite $x^2 + 4x + 4$ as

$$(x + 2)^2 \tag{15}$$

. Therefore, by replacing 14 and 15 in 12, final answer would be

$$\arccos(\frac{4}{\sqrt{x^2 + 4x + 20}}) = \arcsin(\frac{|x + 2|}{\sqrt{x^2 + 4x + 20}}) \tag{16}$$

Solution 7 : D is correct! As we know the domain of the function arcsin is only defined on the interval $[\frac{-\pi}{2} \quad , \quad \frac{\pi}{2}]$. Both $x = \frac{-\pi}{4}$ and $x = \frac{7\pi}{4}$ satisfy the equation

$$\arcsin x = \frac{-\sqrt{2}}{2} \tag{17}$$

However, $x = \frac{7\pi}{4}$ does not belong to domain of arcsin. Therefore it is not acceptable.

Solution 8 : D is correct! For every t one has the identity

$$\sin(-t) = -\sin(t) \tag{18}$$

Hence, $\sin(-t) = -\sin(t) = \frac{-1}{5}$.

Solution 9 : D is correct! Choose the point $A = (1, 0)$ on x-axis. By replacing $x = 1$ in the given line $3x + y = 0$, one has

$$3 + y = 0 \qquad \longmapsto y = -3 \tag{19}$$

Choose the point $B = (1, -3)$ on the given line. In this way one can consider AOB as a right triangle in which $adjacent = 1$ and $opposite = -3$. Then, by using Pythagoras relation one can find the length of hypotenuse according to

$$hyp^2 = adj^2 + opp^2 = 1^2 + (-3)^2 = 10 \qquad \longmapsto hyp = \sqrt{10} \tag{20}$$

Therefore, the value of $\sin(\theta)$ is equal to

$$\sin(\theta) = \frac{opp}{hyp} = \frac{-3}{\sqrt{10}} \qquad or \qquad = \frac{-3\sqrt{10}}{10} \tag{21}$$

Solution 10 : D is correct! Suppose y_1 and y_2 are the length of flagpole under and above our eye sight, respectively. In order to find value of x one can use the definition of tangent function as follow,

$$\tan(50^\circ) = \frac{y_2}{x} \qquad \longmapsto y_2 = \tan(50^\circ)x \tag{22}$$

and

$$\tan(15^\circ) = \frac{y_1}{x} \qquad \longmapsto y_1 = \tan(15^\circ)x \tag{23}$$

We know that the length of flagpole is 80 meters which means that

$$y_1 + y_2 = 80 \tag{24}$$

By replacing 22 and 23 in 24 we have

$$80 = y_1 + y_2 = \tan(15^\circ)x + \tan(50^\circ)x = \quad (\tan(50^\circ) + \tan(15^\circ))x \tag{25}$$

Hence,

$$x = \frac{80}{\tan(50^\circ) + \tan(15^\circ)} \tag{26}$$

Solution 11 : C is correct! In this problem $opp = 1500$ and $hyp = 3000$. Therefore,

$$\sin(\theta) = \frac{1500}{3000} = \frac{1}{2} \qquad \longmapsto \theta = \arcsin(\frac{1}{2}) = 30^\circ \tag{27}$$

Solution 12 : A is correct! We know that in isosceles triangle altitude divides b into two sections with equal lengths. In this case, one can use the relation

$$\tan(\theta) = \frac{altitude}{half \quad of \quad b} = \frac{alt}{10} \tag{28}$$

On the other hand, in an isosceles triangle all angles are equal 60°. Therefore, we have

$$\tan(60^\circ) = \frac{\sin(60^\circ)}{\cos(60^\circ)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \tag{29}$$

By equating relations 28 and 29 one have

$$\sqrt{3} = \frac{alt}{10} \qquad \longmapsto alt = 10\sqrt{3} \tag{30}$$

Solution 13 : B is correct! Recall that a function is odd if and only if $f(-x) = -(f(x))$. Here, if one replaces $-x$ with x in $f(x) = x\cos(x)$, then

$$f(-x) = (-x)\cos(-x) \tag{31}$$

Since, $\cos(-x) = \cos(x)$,

$$f(-x) = (-x)\cos(-x) = -x\cos(x) \qquad \longmapsto f(-x) = -f(x) \tag{32}$$

Therefore, $f(x)$ is odd. By recalling the graph of sin function one realizes that its value alternates between -1 and 1.Hence, Q is also correct. The $y = \tan(x)$ has two consecutive vertical axis in $x = \frac{-\pi}{2}$ and $x = \frac{-\pi}{2}$. Hence, the period of this function is π.

Solution 14 : C is correct! From elementary physics course we know that if an object moves by constant velocity then it satisfies the relation

$$x = vt \tag{33}$$

where, x is distance, v is for velocity and t is time. According to this problem, this ship travels to north east for 2 hours. Therefore, its distance from port is

$$x = vt \qquad \longmapsto x = 10 \times 2 = 20 \tag{34}$$

and after that it ravels to the east for one hour with the same velocity. If we call this length y, then

$$y = vt \qquad \longmapsto y = 10 \times 1 = 10 \tag{35}$$

Now, one can connect origin to the point our ship started to travel east and north axis. It gives us a right triangle. In this case, *opposite* side is the distance from the ship and north axis and x is hypotenuse of this triangle. Hence,

$$the \quad distance \quad from \quad north \quad axis \quad = 20\sin(30^\circ) = 10 \tag{36}$$

The length of adjacent of this triangle is

$$adj = 20\cos(30^\circ) = 10\sqrt{3} \tag{37}$$

At 3 pm this ship has the distance $opp + y = 20$ from north axis. Therefore, one can use Pythagorean theorem in order to find the distance of our ship from port.

$$distance^2 = 20^2 + 10\sqrt{3}^2 = 700 \qquad \longmapsto dis = 10\sqrt{7} \tag{38}$$

Solution 15 : E is correct!

$$\cos(\frac{-19\pi}{6}) = \cos(\frac{19\pi}{6}) = \cos(\frac{18\pi + \pi}{6}) = \cos(3\pi + \frac{\pi}{6}) \tag{39}$$

Since cos is a periodic function with period 2π, one can ignore any multiplication of 2π. Hence, in this case we have

$$\cos(3\pi + \frac{\pi}{6}) = \cos(\pi + \frac{\pi}{6}) = \frac{-\sqrt{3}}{2} \tag{40}$$

Solution 16 : C is correct! Recall that

$$1 + \tan(x)^2 = \frac{1}{\cos(x)^2} \tag{41}$$

Therefore, one can rewrite our expression in the form

$$\ln(1 + \tan(x)^2) + \ln(\cos(x)^2) = \ln(\frac{1}{\cos(x)^2}) + \ln(\cos(x)^2) \tag{42}$$

Now, in order to simplify one can use the fact that

$$\ln(a) + \ln(b) = \ln(ab) \tag{43}$$

In this case, we have

$$\ln(\frac{1}{\cos(x)^2}) + \ln(\cos(x)^2) = \ln(\frac{\cos(x)^2}{\cos(x)^2}) = \ln(1) = 0 \tag{44}$$

Solution 17 : B is correct!

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{8}{5} \tag{45}$$

Therefore,

$$\sin(\theta) = \frac{5}{8} \tag{46}$$

We know that $\sin(\theta)$ has two positive value in quarter 1 and quarter 2. In order to find the unique value of θ we must use other piece of information this problem gives us! Here, we know that $\tan(\theta)$ is negative. But, all of basic trigonometric functions are positive at first quarter. Therefore, θ belongs to second quarter. In order to find the value of $\cos(\theta)$ one can use the relation

$$\cos(\theta) = \sqrt{1 - \sin(\theta)^2} \tag{47}$$

In this case, we have

$$\cos(\theta) = \sqrt{1 - \frac{25}{64}} = \sqrt{\frac{39}{64}} \tag{48}$$

But, since θ belongs to second quarter, the value of $\cos(\theta)$ is negative. Hence

$$\cos(\theta) = -\sqrt{\frac{39}{64}} \qquad or \qquad \frac{-\sqrt{39}}{8} \tag{49}$$

Solution 18 : E is correct! One can use the relation

$$\cos(\frac{\pi}{2} - x) = \sin(x) \tag{50}$$

in order to simplify the given relation. Hence,

$$\sin(x) = \frac{3}{5} \tag{51}$$

Since $\sin(x) > 0$, there are two possible solution for above equation in first and second quarter. Since all simple trigonometric functions are positive in first quarter and by using the assumption that $\cos(x) < 0$, one can realize that x belongs to second quarter and its value is

$$\cos(x) = -\sqrt{1 - \sin(x)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{-4}{5} \tag{52}$$

Hence, $\cot(x)$ equals to

$$\cot x = \frac{\cos(x)}{\sin(x)} = \frac{\frac{-4}{5}}{\frac{3}{5}} = \frac{-4}{3} \tag{53}$$