1) Find whether the following sequences are divergent or convergent. If any of them is divergent, then find the limit.

$\{\frac{(2n)n!}{(n+1)!}\}|_{n=0}^{\infty}$

$\{\frac{\ln(n)}{n}\}|_{n=0}^{\infty}$

$\{(-1)^n \sin(\frac{1}{n})\}|_{n=0}^{\infty}$

2) Suppose, the sequence a_n is given as follow.

$$a_1 = \sqrt{2}, \quad a_2 = \sqrt{2\sqrt{2}}, \quad ... \quad a_n = \sqrt{2\sqrt{2\sqrt{2...}}} \tag{1}$$

As you can see this sequence is monotonic increasing and we have $for \quad all \quad n \quad a_n > 1$. This sequence is convergent. Find

$$\lim_{n \to \infty} a_n \tag{2}$$

Hint: At first glance, it is not possible to solve this problem. However, you can note that when n goes to infinity since $\{a_n\}$ is monotonically increasing, the difference between a_n and a_{n-1} becomes negligible. Therefore, the relation $a_n = \sqrt{2a_{n-1}}$ for large values of n is equivalent to $a_n = \sqrt{2a_n}$. By doing one trick you can find two solutions for the value of a_n and only one of them is acceptable. Choose wisely!

3) Use geometric series in order to find the value of following series.

$$\sum_{n=0}^{\infty} e^n 5^{1-2n} \tag{3}$$