

1) It was shown that the geometric series $\sum_{n=0}^{\infty} x^n$ is convergent for $|x| < 1$ and it converges to $\frac{1}{1-x}$. Use this fact and find a series representation of the function $x^2 \arctan(x)$ and use an appropriate test in order to evaluate the interval of convergence.

2) Find the interval of convergence for the following series.

$$\sum_{n=2}^{\infty} \frac{x^n}{\ln(n)} \quad (1)$$

Hint: $[-1, 1)$ is the solution. Use ratio test to justify this result. Justify the fact that -1 is part of the solution but 1 is not.

3) Find the 5th derivative of the function $\sin(\ln(x + 2))$ at $x = 0$!

Hint: At first glance solving this question is not possible. Recall that the Maclaurin series expansion of $\sin(x)$ has the form $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$. Consider this fact and find a series expansion for the function $\sin(\ln(x + 2))$. Now, compare your result with a Maclaurin series in its standard format $\sum_{n=0}^{\infty} \frac{f^n(0)x^n}{n!}$ where, f^n means n th derivative. Here, we want to find $f^5(0)$. Therefore, $n = 5$. Use this fact in the series you got at first place in order to find your answer.