

1) It was shown that the geometric series $\sum_{n=0}^{\infty} x^n$ is convergent for $|x| < 1$ and it converges to $\frac{1}{1-x}$. Use this fact and find a series representation of the function $\frac{x^2}{(x-2)^2}$ and use an appropriate test in order to evaluate the interval of convergence.

Hint: Remember that $\frac{d}{dx}\left(\frac{1}{x-2}\right) = -\frac{1}{(x-2)^2}$

2) Suppose a parametric curve is given as $(x, y) = (\cos(2t), \sin(2t))$ where, $0 \leq t \leq \pi$.

a) Find $\frac{d^2y}{dx^2}$ at the point $(1, 0)$.

Hint: $\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$ and $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

b) Find the arch length of this curve.

Hint: The arch length of a given parametric curve is given by $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$. Here, $a = 0$ and $b = \pi$.