1. Find the equation of the line passing through point (3,-2) and perpendicular to the line $6y - 3x + 2 = 0$

Solution : First, notice that two lines are perpendicular if the product of their slopes equals -1. Here, we can rewrite $6y - 3x + 2 = 0$ as follow

$$6y - 3x + 2 = 0 \tag{1}$$

$$6y = 3x - 2 \tag{2}$$

$$y = \frac{1}{2}x - \frac{1}{3} \tag{3}$$

Here, the slope of the given line is the coefficient of x which is $\frac{1}{2}$. Hence, if we take the coefficient of our line as m, it must satisfy

$$\frac{1}{2}m = -1 \tag{4}$$

so that these two lines become perpendicular to each other. Therefore, $m = -2$ and the equation of the line passing through (3,-2) with slope-2 is

$$y - (-2) = -2(x - 3) \tag{5}$$

$$y + 2 = -2x + 6 \qquad or \qquad y = -2x + 4 \tag{6}$$

2. Find the value of m for which the lines $mx+3y = -3$ and $2x+(m+1)y = 2$ are parallel.

Solution : Two line are parallel if they have same slopes. Hence, to find m, first we can rewrite each line in its standard form.

$$mx + 3y = -3 \tag{7}$$

$$3y = -mx - 3 \tag{8}$$

$$y = \frac{-m}{3} - 1 \tag{9}$$

and we can rewrite other equation as

$$2x + (m + 1)y = 2 \tag{10}$$

$$(m + 1)y = -2x + 2 \tag{11}$$

$$y = \frac{-2}{m + 1}x + \frac{2}{(m + 1)} \tag{12}$$

Here, the slopes of lines are -2 and $\frac{-2}{(m+2)}$ respectively. Therefore, these two lines are parallel if

$$-2 = \frac{-2}{(m + 2)} \tag{13}$$

$$m + 2 = 1 \qquad (I \quad devide \quad both \quad side \quad of \quad this \quad equation \quad by \quad -2) \tag{14}$$

3.1 Find all real values of x such that $f(x) = 0$.

$$f(x) = x^3 - x^2 - 4x + 4 \tag{15}$$

Solution :

$$x^3 - x^2 - 4x + 4 = 0 \tag{16}$$

$$x^2(x-1) - 4(x-1) = (x^2 - 4)(x-1) = (x-2)(x+2)(x-1) = 0 \tag{17}$$

When the product of two or many real number is zero, then one of them must be zero. Therefore,

$$x - 2 = 0 \qquad x + 2 = 0 \qquad x - 1 = 0 \tag{18}$$

Hence, roots of this equation are 2, -2, and 1.

4.1 For what value of m the set of ordered pairs define a function.
{(6,8) , (-1,4) , (-5,3) , (2m-4,3) , (1,7) }

Solution : A function works as a factory, for every input (the domain of function) there exists an output (the range of function). However, there exist a limitation. Two different elements of domain cannot go to one element of range by a function. Suppose, we expunge the pair (2m-4,3) from our set. Then, it is easy to check that this set is a well defined function. However, if we consider (2m-4,3), then for pairs (2m-4,3) and (-5,3) one faces a problem! Since, for two different values of the domain, namely 2m-4 and -5, we have the same element in rage which is 3. One can overcome this problem if he or she shows that (2m-4,3) and (-5,3) represent the same pair meaning that

$$2m - 4 = -5 \qquad or \qquad m = \frac{-1}{2} \tag{19}$$

5. Find the domain of function $f(x) = \frac{\sqrt{|2x-1|}}{x^2+x+1}$.

Solution : To handle such question, please note that first we consider each nominator and denominator separately and after that we find the intersection of intervals of real numbers which are acceptable for both of them. for $\sqrt{|2x-1|}$, please note that no matter what real number we put in as x, $|2x-1|$ is always a positive number. Therefore, whole real numbers are acceptable in nominator. For denominator whole real numbers are acceptable except those making it zero. To find roots of $x^2 + x + 1$, one can consider the quadratic formula in general format as $ax^2 + bx + c$. Here, $a = 1$, $b = 1$ and $c = 1$. Therefore, one can roots of this equation by using

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{20}$$

and

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{21}$$

However, if one considers a,b, and c as above, then $b^2 - 4ac = (1)^2 - 4(1)(1) = 1 - 4 = -3$. Hence, we have a negative number under the radical sign with an index even meaning that $x^2 + x + 1 = 0$ has no roots. Therefore, all real numbers are acceptable in denominator,too. Then, one can use all real numbers in both nominator and denominator. Hence, the domain of this function is R!