Here is some problems with solutions. This is not complete. Yet, I hope it helps to to do your best on exam.
Find the exact value of

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} \tag{1}$$

Solution: As I mentioned in your class, whenever you are asked to find the exact value of a series you only have two options:
1. Using telescopic method
2. Using geometrical series method
In this case, we need to use telescopic method. To do this, first use the method of partial fractions in order to simplify above series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \sum_{n=1}^{\infty} \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2} \tag{2}$$

where, A,B,C are constants. If you use cover up method you will find that

$$A = \frac{1}{2} \qquad B = -1 \qquad C = \frac{1}{2} \tag{3}$$

Therefore, we have

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \sum_{n=1}^{\infty} \frac{1}{2n} - \frac{1}{n+1} + \frac{1}{2(n+1)} \tag{4}$$

Now, if you recall, for a telescopic series

$$\sum_{n=k}^{\infty} a_n - a_{n-1} \tag{5}$$

the value this series converges to is

$$\sum_{n=k}^{\infty} a_n - a_{n-1} = (\lim_{n\to\infty} a_n) - a_k \tag{6}$$

Now, let's back to our calculation. As you can see, in order to use telescopic method you need an even number of terms. However, in 4, we have an odd number of terms. Here, is a trick that you can apply to other similar problems. We can take $\frac{-1}{n+1}$ in 4 and rewrite it as

$$\frac{-1}{n+1} = -\frac{1}{2}\frac{1}{n+1} - \frac{1}{2}\frac{1}{n+1} \tag{7}$$

By using this tiny trick we can rewrite 4 as

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \sum_{n=1}^{\infty} \frac{1}{2(n+2)} - \frac{1}{2(n+1)} - \frac{1}{2(n+1)} + \frac{1}{2n} \tag{8}$$

Then, we can split the above series and simplify it as

$$\sum_{n=1}^{\infty} \frac{1}{2(n+2)} - \frac{1}{2(n+1)} - \sum_{n=1}^{\infty} \frac{1}{2(n+1)} - \frac{1}{2n} \tag{9}$$

These two series have the same format of telescopic series. Therefore, we can apply 6 to each of them individually

$$\sum_{n=1}^{\infty} \frac{1}{2(n+2)} - \frac{1}{2(n+1)} = (\lim_{n\to\infty} \frac{1}{2(n+2)}) - \frac{1}{2(1+1)} = \frac{-1}{4} \tag{10}$$

For the second one we have

$$\sum_{n=1}^{\infty} \frac{1}{2(n+1)} - \frac{1}{2n} = (\lim_{n\to\infty} \frac{1}{2(n+1)}) - \frac{1}{2(1)} = \frac{-1}{2} \tag{11}$$

Therefore, the final solution is equal to

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \frac{-1}{4} - (\frac{-1}{2}) = \frac{1}{4} \tag{12}$$

As an exercise, try to find the convergent value of

$$\sum_{n=4}^{\infty} \frac{1}{n^3 - n} \tag{13}$$

Problem 2: Determine if the following series

$$\sum_{n=4}^{\infty} \frac{n^n}{n!} \tag{14}$$

is convergent or divergent. Solution: If you recall the growth rate for different functions we have

$$\ln(n)^p < n^q < a^n < n! < n^n \tag{15}$$

where, p,q are positive numbers and $a > 1$. Now, in this case we can consider

$$n^n > n! \qquad \to \qquad \frac{n^n}{n!} > 1 \tag{16}$$

This is equivalent to say that

$$\lim_{n\to\infty} \frac{n^n}{n!} \neq 0 \tag{17}$$

and as a consequence, this series is divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n)^2}{2+n} \tag{18}$$

Solution: Please note that for any value of n we have

$$0 < sin^2(n) < 1 \tag{19}$$

Therefore, we can use both comparison and alternating series criteria. First, note that for every n

$$\frac{\sin(n)^2}{2+n} < \frac{1}{n} \tag{20}$$

hence,

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n)^2}{2+n} < \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \tag{21}$$

Now, we can use alternating series criteria. First, $\frac{1}{n}$ is decreasing. Second $\frac{1}{n}$ is positive for every n. Third, $\lim_{n\to\infty} \frac{1}{n} = 0$. Therefore, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is convergent and as a result, the series $\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n)^2}{2+n}$ is convergent.

$$\sum_{n=1}^{\infty} \frac{1+2^n}{n^2+2^n} \tag{22}$$

Solution: Divide both numerator and denominator by 2^n. Here, we have

$$\sum_{n=1}^{\infty} \frac{1+2^n}{n^2+2^n} = \sum_{n=1}^{\infty} \frac{(\frac{1}{2})^n + 1}{\frac{n^2}{2^n} + 1} \tag{23}$$

Now, since the rate of growth of n^2 is less than 2^n, then

$$\lim_{n\to\infty} \frac{n^2}{2^n} = 0 \tag{24}$$

Hence, we can say

$$\lim_{n\to\infty} \frac{1+2^n}{n^2+2^n} = \lim_{n\to\infty} \frac{(\frac{1}{2})^n + 1}{\frac{n^2}{2^n} + 1} = \lim_{n\to\infty} \frac{(\frac{1}{2})^n + 1}{\frac{n^2}{2^n} + 1} = \frac{0+1}{0+1} = 1 \tag{25}$$

Since $\lim_{n\to\infty} \frac{(\frac{1}{2})^n+1}{\frac{n^2}{2^n}+1} \neq 0$, therefore, this series is divergent.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin(\frac{1}{n}) \tag{26}$$

Solution: For the case that the phase part,θ, of $\sin(\theta)$ and $\tan(\theta)$ is very close to zero, then the following equations are true

$$\sin(\theta) = \theta \qquad \tan(\theta) = \theta \tag{27}$$

Now, back to this example, since $n \to \infty$, then the phase of sin function namely $\frac{1}{n}$ approximates zero. Therefore, equations 27 are valid and we can rewrite the given series as

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin(\frac{1}{n}) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}} \tag{28}$$

Now, as you recall the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \tag{29}$$

is convergent if $p > 1$ and divergent if $p \leq 1$. In our case $p = \frac{3}{2}$ and as a result this series is convergent. As an exercise, determine if the series $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{5}}} \tan(\frac{1}{\sqrt[5]{n^2}})$ is divergent or convergent.