

1) Find the interval of convergence for the following series.

$$\sum_{n=2}^{\infty} \frac{x^n}{n \ln(n)} \tag{1}$$

Solution:  $[-1, 1)$

$$\sum_{n=2}^{\infty} \frac{n^2 x^{3n}}{\ln(n+1)} \tag{2}$$

Solution:  $(-1, 1)$ .

2) Find the 12th derivative of the function  $x^2 e^{2x}$  at  $x = 0$ .

Solution:  $\frac{2^{10}}{10!}$ .

3) Find the 6th derivative of  $\ln(1+x)$  at  $x=0$  by comparing the Mac-Laurin series with Taylor's expansion of the given function.

Solution:  $-5!$ .

4) Evaluate the exact value of the following series.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \pi^{2n}}{4^{2n} 2n!} \tag{3}$$

Solution: Consider  $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n!}$ . Compare the given series with series expansion of  $\cos(x)$ . The answer is  $-\frac{\sqrt{2}}{2}$ .

5) Find the series expansion for the following functions.

$$\frac{1}{4-x^2} \tag{4}$$

Solution: Use the partial fraction method and then use the fact that  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$  in order to find a series expansion of the given function.

$$\int \arctan(x^2) \tag{5}$$

Solution: First, evaluate the derivative of the function  $\arctan(x^2)$ . Then, find a series expansion for the derivative and finally use the fact that the integral of the derivative of a given function is equal to itself.

$$\frac{x^3}{(x-2)^2} \tag{6}$$

Solution: Use the fact that  $\frac{d}{dx} \left( \frac{1}{x-2} \right) = \frac{-1}{(x-2)^2}$ .

6) Evaluate the length of the parametric curve  $(e^t \cos(t), e^t \sin(t))$  from  $t = 0$  to  $t = 1$ .

Solution: Evaluate  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$  and then use the relation  $\int \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$ .

7) Find the equation of the tangent and normal line on the parametric curve  $c(t) = (e^t + e^{-t}, t^2)$  at the point  $(2, 0)$  and also evaluate the concavity of this function at this given point.

Solution: First, find for what value of  $t$   $(e^t + e^{-t}, t^2) = (2, 0)$ . Then, use relations  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$  in order to find the equation for tangent and normal line and  $\frac{d^2y}{dx^2} = \frac{d}{dt}(\frac{dy}{dx})$  in order to find the concavity.

8) Find the area out side the curve  $r_1(\theta) = 1 + \sin(\theta)$  and inside  $r_2 = \frac{1}{2}$ .

Solution: First, find the interval at which  $r_1 = r_2$ . Then, use the relation  $\frac{1}{2} \int (r_1^2(\theta) - r_2^2) d\theta$

9) Find the equation of given polar functions in Cartesian plane.

$$r = 2 \sin(\theta) \qquad \tan(\theta) = 2 \qquad \sec(\theta) = 3r \qquad (7)$$

Solution: Use relations  $x^2 + y^2 = r^2$  and  $x = r \cos(\theta)$  and  $y = r \sin(\theta)$  in order to solve this problem.

10) Find the series expansion for the following function.

$$\int \frac{e^{3t} - 1 - 3t}{t^2} dt \qquad (8)$$

Solution:  $\sum_{n=2}^{\infty} \frac{3^n t^{n-1}}{(n-1)n!} + C$