1) Find the interval of convergence for the following series.

\[ \sum_{n=2}^{\infty} \frac{x^n}{n \ln(n)} \]  
(1)

Solution: \([-1, 1)\)

\[ \sum_{n=2}^{\infty} \frac{n^2 x^{3n}}{\ln(n + 1)} \]  
(2)

Solution: \((-1, 1)\).

2) Find the 12th derivative of the function \(x^2 e^{2x}\) at \(x = 0\).
Solution: \(\frac{2^{10}}{10!}\).

3) Find the 6th derivative of \(\ln(1 + x)\) at \(x = 0\) by comparing the Mac-Laurin series with Taylor’s expansion of the given function.
Solution: \(-5!\).

4) Evaluate the exact value of the following series.

\[ \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \pi^{2n}}{4^{2n} 2n!} \]  
(3)

Solution: Consider \(\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n!}\). Compare the given series with series expansion of \(\cos(x)\). The answer is \(-\frac{\sqrt{2}}{2}\).

5) Find the series expansion for the following functions.

\[ \frac{1}{4 - x^2} \]  
(4)

Solution: Use the partial fraction method and then use the fact that \(\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n\) in order to find a series expansion of the given function.

\[ \int \arctan(x^2) \]  
(5)

Solution: First, evaluate the derivative of the function \(\arctan(x^2)\). Then, find a series expansion for the derivative and finally use the fact that the integral of the derivative of a given function is equal to itself.

\[ \frac{x^3}{(x - 2)^2} \]  
(6)

Solution: Use the fact that \(\frac{d}{dx} \left(\frac{1}{x-2}\right) = \frac{-1}{(x-2)^2}\).
6) Evaluate the length of the parametric curve \((e^t \cos(t), e^t \sin(t))\) from \(t = 0\) to \(t = 1\).
Solution: Evaluate \(\frac{dx}{dt}\) and \(\frac{dy}{dt}\) and then use the relation \(\int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt\).

7) Find the equation of the tangent and normal line on the parametric curve \(c(t) = (e^t + e^{-t}, t^2)\) at the point \((2, 0)\) and also evaluate the concavity of this function at this given point.
Solution: First, find for what value of \(t\) \((e^t + e^{-t}, t^2) = (2, 0)\). Then, use relations \(\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}\) in order to find the equation for tangent and normal line and \(\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx}\right) \frac{dt}{dx}\) in order to find the concavity.

8) Find the area out side the curve \(r_1(\theta) = 1 + \sin(\theta)\) and inside \(r_2 = \frac{1}{2}\).
Solution: First, find the interval at which \(r_1 = r_2\). Then, use the relation \(\frac{1}{2} \int (r_1^2(\theta) - r_2^2) d\theta\)

9) Find the equation of given polar functions in Cartesian plane.

\[
r = 2 \sin(\theta) \quad \tan(\theta) = 2 \quad \sec(\theta) = 3r \quad \tag{7}
\]
Solution: Use relations \(x^2 + y^2 = r^2\) and \(x = r \cos(\theta)\) and \(y = r \sin(\theta)\) in order to solve this problem.

10) Find the series expansion for the following function.

\[
\int \frac{e^{3t} - 1 - 3t}{t^2} dt \quad \tag{8}
\]
Solution: \(\sum_{n=2}^{\infty} \frac{3^n n^{n-1}}{(n-1)!} + C\)