Numerical Analysis
Project 1:

[**Root Finding Methods**] The goal is to find the points at which $e^{-x/5} = \sin(x)$ on the interval $[0, 10]$.

1. [**Visual Inspection**] Use matlab to plot the functions $e^{-x/5}$ and $\sin(x)$. From this make your first approximations to the points where they are equal.

2. [**Bisection Method**] Write a short matlab program which will implement the bisection method, given initial intervals $[a_k, b_k]$. Use the results from Problem 1 to implement the program, and find the approximations to the points where $f(x) = e^{-x/5} - \sin(x) = 0$. Let the midpoint method run until $f(x) < 10^{-7}$. Count the number of steps that the midpoint method takes for each point.

3. [**Newton's Method**] Write a short matlab program which will implement Newton's method, given an initial starting point, such as $b_k - a_k$ from above. Let the midpoint method run until $f(x) < 10^{-7}$. Count the number of steps that the Newton's method takes for each point. Compare the speed to Newton's method to that of the midpoint method.

4. [**Newton's Method Part 2**] a) Use Newton's method to compute the solution to $(x-3)^4 \sin(x)$ using $x_0 = 2$. Note its convergence rate. b) Use the altered Newton's method to compute this and note it's convergence rate.

[**Interpolation and Approximation Methods**] Compare and Contrast the following Interpolation/Approximation Methods.

4. [**Lagrange Interpolation**] Interpolate

$$f(x) = \frac{1}{1 + x^2}$$

at evenly spaced points on the interval $[-5, 5]$, with Lagrange polynomials of order $n = 5, 10, 20$. Does the approximation get better?

5. [**Piecewise Linear Interpolation**] Interpolate

$$f(x) = \frac{1}{1 + x^2}$$

at evenly spaced points on the interval $[-5, 5]$ using piecewise linear interpolation with the same points as in Problem 4. Does the approximation get better with more points?

6. [**Raised Cosine Interpolation**] Interpolate

$$f(x) = \frac{1}{1 + x^2}$$

at evenly spaced points on the interval $[-5, 5]$ using a raised cosine basis function and the same points as in Problem 4. Does the approximation get better with more points? Does this seem better than the result from Problem 4 and 5?

7. [**Least Squares Approximation**] Approximate

$$f(x) = \frac{1}{1 + x^2}$$

using least squares approximation with polynomials of order $n = 5, 10, 20$ on $[-5, 5]$. Does the approximation get better higher order polynomials? Does this seem better than the result from Problem 4, 5, and 6?

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Project 1: Bonus 5 pts

1. Square Root Calculator: Write a program which will eventually converge at a quadratic rate and calculate the square root of any number, using only addition, multiplication, and division. Test this program on numbers from 10-10000. Explore options for picking a starting point... perhaps for Newton's method to finish. Give a succinct and detailed report of your explorations. Demonstrate that it converges at a quadratic rate.

2. Altered Newton's Method: Write a program which will alter Newton's method when a multiple zero is involved. Use the examples of $(x-4)^2 \sin(x)$ and $(x-4)^3 \sin(x)$ as test cases. The algorithm should be able to identify the level of the zero (p = 2 or 3 in the previous examples), and alter its approach for a quadratic convergence rate. Demonstrate that it converges quadratically in both cases.