Fourier Analysis
Project 1:

The project must prepared in an orderly and professional manner. Handwritten calculations are OK, but you must present them in an orderly fashion along with the matlab graphs.

1. Compute the Fourier series for the function $f(t) = t$ which converges a) on $[-\pi, \pi]$, and b) on $[-2\pi, 2\pi]$. Plot the approximations using 5,10, and 15 terms on $[-4\pi, 4\pi]$. These are two different series and should look different.

2. Figure out the necessary cosine, and sine terms, and the appropriate coefficient formulas to approximate a function on the interval $[1, 3]$.

3. Using the result from Problem 2 above, calculate the first few terms of the expansion of $f(t) = t$ on $[1, 3]$. Plot this result on $[-1, 5]$.

4. Verify that the Fourier Isometry holds on $[-\pi, \pi]$ for $f(t) = t$. To do this, a) calculate the coefficients of the orthogonal Fourier series from the orthogonal series representation, b) calculate the sum of the squared coefficients, and c) Calculate the norm of the function as $\int_{-\pi}^{\pi} |f(t)|^2 dt$. They must be equal. How many terms in the Fourier series are necessary to have the isometry be under 5%? How many until you are under 3%, or 1%.

5. **Gibbs Ringing:** Calculate the Fourier series for $\chi_{1/2}(t)$ on $[-1, 1]$, and plot the first 50 terms. Find the maximum value of the series with 30, 40, 50, and 100 terms. Guess at the limit of this maximum value. Find the location of the maximum value on the right of 0, with 30, 40, 50, and 100 terms. Where is the maximum value headed? Finally, estimate the squared error $\int_{-1}^{1} |\chi_{1/2}(t) - S_n(\chi_{1/2})(t)|^2 dt$ and the series on $[-1, 1]$ with the same number of terms (do this numerically).

6. **Sine and Cosine Series:** Calculate the sine and cosine series for $f(t) = t$ on $[0, 1]$. a) Plot the first 30 terms of these series on $[-3, 3]$. Estimate the error between both series and the function on $[0, 1]$ after 30 terms. b) How many more terms of the sine series are necessary to achieve the same error as was achieved with the cosine series and 30 terms?

7. **Sine and Cosine Series:** Plot the first 30 terms of the derivatives of both the sine and cosine series in the above problem. What do you observe? Do both of them converge?

8. **Convolution:** Calculate the Fourier series for the hat function $f(t) = 2\pi - |t|$ on $[-2\pi, 2\pi]$ in two ways. a) By calculating its Fourier series directly. b) By calculating the Fourier series for $\chi_\pi(t)$ and using the convolution theorem. They must be equal! Use trig identities...