Numerical Analysis
Project 2:
Fall 2017

[Numerical Integration] Find \( \int_0^6 f(x) \, dx \) with the function \( f(x) = \frac{2x}{1 + x^2} \) using the following methods. Please analyze the convergence as \( h \to 0 \) for \( h = .1, .01, .001 \) until you have a viable answer. Write matlab codes which will allow you to change the stepsize. Show graphs of your numerical outputs for comparison.

1. [Riemann Sums] The old standard.


3. [Simpson’s Rule] Does this rule do a better job on either of the functions....? We hope so.. report the actual results.

4. [Comparison] Plot the results on one common graph and compare them.

[Numerical Differentiation] Use the function \( \ln(1 + x^2) \) as a test case to explore the following three possible differentiation techniques on \([0, 6]\). Use the values of \( h = .1, .01 \). Remember that the methods require special exceptions on the endpoints.

5. [Backwards, Forwards, and Symmetric] Compute the first two, and use them to get the third. Plot them on common graphs for comparison. Analyze the results.

[Numerical ODE Solving Routines] Consider the following initial value ODE problems: 1) \( y' = 3y, \ y'(0) = 1 \), on \([0, 3]\), and 2) \( y' = 1/(1 + x^2) - 2y^2 \), with \( y(0) = 0 \), which has the solution \( y(x) = x/(1 + x^2) \), on \([0, 10]\). Utilize the following methods, for various step sizes \( h = .1, .01, .001 \). Use the maximum error as the test of accuracy.

6. [Euler’s Method] As simple as it gets.


8. [Trapezoid Method] Depending on the problem, either solve to get an implicit method, or use iteration to approximate.

9. [Comparison] Compare the methods. Which seems best?

[Bonus] (a) Convert the second order differential equation, \( y'' + 25y = 0, \ y(0) = 0, \ y'(0) = 1 \), into a first order system, (b) Solve the system with the three methods in 6-9 above, and (c) Compare the results for \( h = .1, .01, .001 \).