Numerical Analysis
Project 2:
Fall 2017

[**Numerical Integration**] Find $\int_0^6 f(x)dx$ with the function $f(x) = 2x/(1+x^2)$ using the following methods. Please analyze the convergence as $h \to 0$ for $h = .1, .01, .001$ until you have a viable answer. Write matlab codes which will allow you to change the stepsize. Show graphs of your numerical outputs for comparison.

1. [**Riemann Sums**] The old standard.

2. [**Trapezoid Rule**] The next best thing.

3. [**Simpson's Rule**] Does this rule do a better job on either of the functions....? We hope so.. report the actual results.

4. [**Comparison**] Plot the results on one common graph and compare them.

[**Numerical Differentiation**] Use the function $ln(1+x^2)$ as a test case to explore the following three possible differentiation techniques on $[0,6]$. Use the values of $h = .1, .01$. Remember that the methods require special exceptions on the endpoints.

5. [**Backwards, Forwards, and Symmetric**] Compute the first two, and use them to get the third. Plot them on common graphs for comparison. Analyze the results.

[**Numerical ODE Solving Routines**] Consider the following initial value ODE problems: 1) $y' = 3y$, $y(0) = 1$, on $[0,3]$, and 2) $y' = 1/(1+x^2) - 2y^2$, with $y(0) = 0$, which has the solution $y(x) = x/(1+x^2)$, on $[0,10]$. Utilize the following methods, for various step sizes $h = .1, .01, .001$. Use the maximum error as the test of accuracy.

6. [**Euler's Method**] As simple as it gets.

7. [**Midpoint Method**] A little bit more adventurous.

8. [**Trapezoid Method**] Depending on the problem, either solve to get an implicit method, or use iteration to approximate.

9. [**Comparison**] Compare the methods. Which seems best?

[**Bonus**] (a) Convert the second order differential equation, $y'' + 25y = 0$, $y(0) = 0$, $y'(0) = 1$, into a first order system, (b) Solve the system with the three methods in 6-9 above, and (c) Compare the results for $h = .1, .01, and .001$.