1. Let the non-negative integrable function \( f : [a, b] \to \mathbb{R} \) satisfy \( \int_a^b f = 0 \). Prove that if \( \varepsilon > 0 \) then there exists a positive-length closed interval \( I \subseteq [a, b] \) on which the supremum of \( f \) is strictly less than \( \varepsilon \).

2. Let the non-negative integrable function \( f : [a, b] \to \mathbb{R} \) satisfy \( \int_a^b f = 0 \). Prove that the zero-set \( \{ x : f(x) = 0 \} \) of \( f \) is dense in \( [a, b] \) by a ‘nested-intervals’ argument using a sequence \( \varepsilon_n \downarrow 0 \).