1. For $a \in \mathbb{D}$ (the open unit disc) and $(1/\bar{a}) \neq z \in \mathbb{C}$ define

$$f_a(z) = \frac{z - a}{1 - \bar{a}z}.$$

Prove (in any convenient order) that: (i) $f_a$ is injective and continuous on $\overline{\mathbb{D}}$; (ii) $f_a$ is holomorphic on an open nhood of $\overline{\mathbb{D}}$; (iii) $f_a(\mathbb{D}) = \mathbb{D}$ and $f_a(\overline{\mathbb{D}}) = \overline{\mathbb{D}}$; (iv) $(f_a|_{\overline{\mathbb{D}}})^{-1} = f_{-a}|_{\overline{\mathbb{D}}}$.

2. Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$, the indicated power series having radius of convergence $R > 0$. Prove that if $0 \leq r < R$ then (Parseval)

$$\int_{-\pi}^{\pi} |f(re^{i\theta})|^2 d\theta = 2\pi \sum_{n=0}^{\infty} |a_n|^2 r^{2n}.$$

3. Again let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ but now assume specifically that $R = \infty$. Prove that if this function $f : \mathbb{C} \to \mathbb{C}$ is bounded then it is constant. [This is the first appearance of the famous Liouville theorem.]