Modern Analysis 2  
Homework 01

1. Let \( f : [0, 1] \rightarrow \mathbb{R} \) be defined by \( f(0) = 1 \), by \( f(t) = 1/n \) whenever \( t = m/n \) is a nonzero rational (expressed in lowest terms) and \( f(t) = 0 \) when \( t \) is irrational. Is \( f \) Riemann-integrable over \([0, 1]\)? If so, what is the value of its integral?

2. Let \( f : [0, 1] \rightarrow \mathbb{R} \) be Riemann-integrable. Prove that if the Riemann integral \( \int_0^1 f \) is strictly positive then there exist \( \delta > 0 \) and a nonempty open interval \( I \subseteq [0, 1] \) such that \( f \geq \delta \) throughout \( I \).